A Method to Complement Incomplete Hesitant Multiplicative Preference Relation

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ABSTRACT Preference relations are obtained in the decision-making process by comparing different alternatives/criteria by the decision maker. The decision maker may provide his/her judgment by means of hesitant multiplicative preference relation (HMPR) for hesitancy and uncertainty in the scale of 1/9 – 9. Limitation of time, experience and lack of the experts' professional knowledge lead to form an incomplete hesitant multiplicative preference relation. In this contribution, we develop a method to complete HYMPRs. The study of the consistency of HMPRs is an essential feature to keep away from the confusing solution. A new definition of multiplicative transitive property of HMPR has given that preserve the hesitancy property and is used to construct the complete HMPR from incomplete one. An optimization model is developed to minimize the error. The satisfaction degree and the acceptably consistent of complete HMPR is also checked. The whole procedure is explained with a suitable example.

**Keywords:** Multiplicative preference relation, Hesitant multiplicative preference relation, Hesitant multiplicative element, Satisfaction degree, Acceptably consistent.

1 Introduction
In the decision-making process, the opinion of experts results in preference relations, are given by comparison matrices are important tools that used to express the experts' preference value of alternative/criteria. Different type of preference relation have been developed such as fuzzy preference relation (FPR)(Orlovsky 1978, Tanino 1984), multiplicative preference relation (MPR)(Satty 1980),intuitionistic fuzzy preference relation (I-FPR)(Xu 2007, Xu 2013), intuitionistic multiplicativepreference relation (I-MPR)(Xia et.al 2013), theinterval-valued intuitionistic fuzzy preference relation (IVI-FPR)(Xu and Cai 2009, Xu and Yager 2009), interval-valued intuitionistic multiplicativepreference relation (IVI-MPR)(Yu et. al 2013),linguistic preference relation (Herrera-riedma et.al 2007) etc. In case of fuzzyscenario the scale of membership is taken 0-1ratio scale and in case of MPR the ratio scale is 1/9 - 9.

1.1 Research issues and need for the present work
In real life scenario, it is not always possible for a decision maker to provide a single preference value but may comfortable is giving preference in terms of hesitancy which have more possible values andhesitant preference relation can be constructed. Hesitant preference relations are of two types. Onen is hesitant fuzzy preference relation (HFPR) inwhich the preferences are given in the scale 0-1.For ready reference one may refer to(Zhu and Xu 2013) and many more. In HFPR, information is expressed using the scale 0-1 that is equivalently distributed around 0.5. The grade of preferencesis asymmetically distributed aroundsome value. In 2013 Xia et.al specified the distance between the good information grades shouldbe greater than ones between the bad informationgrades. To express the multiplicative preference relation (MPR), Satty (1980) developed 1-9 scalesto address such problem which has been applied in different research areas. In decision scenario, sometimes expert may not prefer the ratio scale0-1, but he/she would use Satty's ratio scale (as inmultiplicative preference relations) to provide theinformation that thealternatives \( x_i \) is better than \( x_j \), for example, few decision makers in the decision problem give 1/3, some give 4, and some give 5, then the decision information to represent thealternative \( x_i \) superior to \( x_j \), can be considered as a set of possible value as \{1/3,4,5\} which is calledhesitant multiplicative element (HME)(Xia and Xu 2013). Xia and Xu (2013) used the 1-9 scale to propose the theory of hesitant multiplicativepreference relation (HMPR). In this work, we concentrate only on hesitant multiplicative preference relation (HMPR).

A complete preference relation of order \( n \times n \) need \( \frac{n(n-1)}{2} \) judgements in its complete upper or lowertriangular section. Sometimes the time restraining situations and absence of sufficient knowledgemake it very difficult to obtain a complete HMPR in many practical situations, such as medical diagnosis, personal examination. These situations result in an incomplete HMPR where some
1.3 Organization of the paper
The setup of the paper is as follows. In section 2, the rudiments of MPRs and HMPRs are defined briefly. Multiplicative transitive property for HMPRs defined that is used to construct complete HMPRs from the incomplete one discussed in section 3. In section 4, deviation model is defined to evaluate the missing valueand satisfaction degree also checked. An example is given to illustrate the whole procedure followed by the concluding remarks.

2 Preliminaries
In this section, we discuss some basic concept of multiplicative preference relation (MPR), hesitant multiplicative set and hesitant multiplicative preference relations.

2.1 Multiplicative preference relations
Consider $X = \{x_1, x_2, ..., x_n\}$ as a discrete set of alternatives where a decision maker has to compare in pairs, and $N = \{1, 2, ..., n\}$ be the set of indices.

Definition 1 (Satty 1980, Satty 1977) A Multiplicative preference relation (MPR) over $X$ is defined by $P = [p_{ij}]_{n \times n}$ where $p_{ij}$ is the multiplicative preference value to which an alternative $x_i$ is preferred to an alternative $x_j$ satisfying $p_{ij}p_{ji} = 1, p_{ij} \geq 0, \forall i, j \in N$.

$p_{ij} \in \left[\frac{1}{9}, 9\right], \forall i, j = 1, 2, ..., n$ i.e. multiplicative preference value measured using 1-9 ratio scale. The preference value $p_{ij}$ with value 1 specify indifference between $x_i$ and $x_j$, $p_{ij} = 9$ specify that the alternative $x_i$ is absolutely preferred to $x_j$,

$p_{ij} = 1/9$ specify that $x_j$ is absolutely preferred to $x_i$, $p_{ij} > 1$ indicates that the preference value of the alternatives $x_j$, over $x_i$ and $p_{ij} < 1$ specify the preference value of the alternatives $x_j$ over $x_i$.

Definition 2 (Satty 1980, Satty 1977) A MPR $P = (p_{ij})_{n \times n}$ satisfying the multiplicative transitivity property

$p_{ij} = p_{ik}p_{kj} \forall i, j, k = 1, 2, ..., n$

is called multiplicative consistent.

If $P = (p_{ij})_{n \times n}$ is consistent MPR, then it should satisfy the property as

$p_{ij} = \frac{w_i}{w_j}, i, j \in N \tag{1}$

Where $w = (w_1, w_2, ..., w_n)$ is the priority vector of the objectives satisfying the $\sum_{i=1}^{n} w_i = 1, w_i \geq 0, i \in N$.

2.2 Hesitant Multiplicative set
Xia and Xu (2013) defined the concept of hesitant multiplicative set in which elements are the set of possible values.

Definition 3 (Xia and Xu 2013) A hesitant multiplicative set on $X$ is defined by $M = \{\{x, b_M(x)\} | x \in X\}$, where $b_M(x)$ is a set of some values that lies between 1/9 to 9. For convenience $b_M(x) = b_x$ is called the hesitant multiplicative element (HME).

Given an HME $b_e = \{b^e_s \mid s = 1, 2, ..., l_{b_e}\}$
where \( l_{b_{ij}} \) denotes the number of elements in the HME. \( \sigma(s) \) denotes the position of HME.

Let \( b_{e}^{\sigma(s)}, b_{e1}^{\sigma(s)}, b_{e2}^{\sigma(s)} \) are the \( s^{th} \) position value of HME \( b_e, b_{e1}, b_{e2} \) respectively. Additionally, suppose that \( l_{b_{e1}} = l_{b_{e2}} = l \), or else we can extend the smaller one by adding the numerical values given as the following definition.

**Definition 4** (Zhang et al. 2014) Let \( b_e^{+} \) and \( b_e^{-} \) are the maximum and minimum elements in HME \n
\[ b_e = \{ b_e^{\sigma(s)} | s = 1, 2, ..., l_{b_{e}} \} \]

be an optimized parameter that lies between 0 and 1 which can be chosen by the decision makers according to their own risk preferences, then \( (b_e^{+})^\delta \times (b_e^{-})^{1-\delta} \), is called an added element that is denoted by \( b_e \). Particularly, the added element \( b_e^{+} \) and \( b_e^{-} \) respectively derived from the conditions \( \delta = 1 \) and \( \delta = 0 \) that corresponds to the optimism and pessimism rules are taken by the decision makers.

**2.3 Hesitant Multiplication preference relation**

Xia and Xu (2013) developed the concept of HMPRs on the basis of hesitant multiplicative elements (HMEs) and multiplicative preference relations (MPRs) given as follows

**Definition 5** (Xia and Xu 2013) A hesitant multiplicative preference relation \( B_R \) over \( X \) is defined by

\[ B_R = \{ b_{eij} \}_{n \times n} \] where \( b_{eij} = \{ b_{eij}^{\sigma(s)} | s = 1, 2, ..., l_{b_{eij}} \} \) is a HME indicating all the possible preference value to which an alternative \( x_i \) is prefer to alternative \( x_j \)

\[ b_{eij}^{\sigma(s)} \times b_{eij}^{\sigma(l_{b_{eij}} - s + 1)} = 1, \quad b_{eij} = \{ 1 \}, \quad l_{b_{eij}} = l_{b_{eij}}, \forall i, j \in N. \]

Where \( b_{eij}^{\sigma(s)} \) is the \( s^{th} \) position element in \( b_{eij} \).

Xia and Xu (2013) mentioned that HME is assumed in increasing order. But it may be the case that some decision maker gives their preference in decreasing order that also satisfies the hesitancy property equation 2. Also, Zhang and Guo (2016) proved following theorem

**Theorem 1:** Let \( B = \{ b_{eij} \}_{n \times n} \), where \( b_{eij} = \{ b_{eij}^{\sigma(s)} | s = 1, 2, ..., l_{b_{eij}} \} \) be an HMPR. It can be transfer into HFPR

\[ H = \{ h_{eij} \}_{n \times n}, \quad h_{eij} = \{ h_{eij}^{\sigma(s)} | s = 1, 2, ..., l_{h_{eij}} \} \] by using the transformation \( h_{eij}^{\sigma(s)} = \frac{b_{eij}^{\sigma(s)}}{1 + b_{eij}^{\sigma(s)}} \). Then transfer the HFPR \( H \) into the HMPR \( B' = \{ b_{eij}^{'} \}_{n \times n} \) by using the transformation \( b_{eij}^{'} = \frac{b_{eij}^{\sigma(s)}}{1 - b_{eij}^{\sigma(s)}} \), where

\[ b_{eij}^{'} = \{ b_{eij}^{\sigma(s)} | s = 1, 2, ..., l_{b_{eij}^{'}} \} \] then \( B = B' \). In our case we are taking the HME into increase/decrease that also satisfies the theorem 1.

In this paper, we are not considering the case that HMEs are arranged in increasing order always. For example, let the HMPR \( B_R \) as shown below

\[
B_R = \begin{bmatrix}
  \{1\} & \{1,1\} & \{2,4\} & \{5\} \\
  \{5,6,7\} & \{1\} & \{1,1\} & \{5,6,7\} \\
  \{1\} & \{4',2\} & \{2,3,4\} & \{1\} & \{4,5\} \\
  \{1\} & \{1\} & \{1,1\} & \{5,4\} & \{1\} \\
\end{bmatrix}
\]

Since the length of the preference degree of alternative \( x_i \) over \( x_j \) in HMPR matrix are not same, based on definition 4, Zhang et. al (2014) used the optimized parameter \( \delta \) to add some elements to an HMPR and finally obtained a normalized hesitant multiplicative preference relation (N-HMPR) defined as follows.

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Definition 6 (Zhang et al. 2014) Let \( b^+_{eij} \) and \( b^-_{eij} \) be the maximum and minimum elements of HMPR, \( B_R = (b_{eij})_{n \times n} \) be respectively, for \( i, j \in N \), and let \( \delta \) be an optimized parameter.

According to definition 4 if we add some elements \( \bar{b}_{eij} = (b^+_{eij})^\delta \times (b^-_{eij})^{1-\delta} \) to \( b_{eij} \) if \( i \leq j \), and add some elements \( \bar{b}^{-1}_{eij} = (b^+_{eij})^{1-\delta} \times (b^-_{eij})^\delta \) then the normalized hesitant multiplicative preference relation (N-HMPR) \( \bar{B}_R = \{\bar{b}^{\sigma(s)}_{eij}| s = 1, 2, ..., l_{\bar{b}_{eij}}\} \) satifying

\[
\begin{align*}
\bar{b}^{\sigma(s)}_{eij} &= \max \{|b_{eij}| i, j = 1, 2, ..., i \neq j \}, \\
\bar{b}_{eij} = (\bar{b}_{eij})^{\sigma(l_{\bar{b}_{eij}} - s + 1)} &= 1, \bar{b}_{ii} = \{1\}. \quad (3)
\end{align*}
\]

Where \( \bar{b}^{\sigma(s)}_{eij} \) and \( \bar{b}^{\sigma(s)}_{eij} \) is the \( s^{th} \) position element in \( \bar{b}_{eij} \) and \( \bar{b}_{eij} \), respectively.

Sometimes it is very difficult to obtain a complete preference relation especially for the preference relation with higher order. It may be the case that an expert is unable to express his/her opinion over other pairs of alternatives, due to the limitation of decision makers’ proficient knowledge, experience or time pressure, the provided preference value in HMPR becomes incomplete. Then the incomplete HMPR is constructed in which some elements are missing. We have defined the incomplete HMPR.

Definition 7 (Incomplete HMPR) The HMPR \( B_R = (b_{eij})_{n \times n} \) where \( \{b^{\sigma(s)}_{eij}| s = 1, 2, ..., l_{b_{eij}}\} \) is called an incomplete HMPR, if some elements in it are missing that is denoted by “*” and the other elements are given by the decisionmakers should satisfy the condition of definition 5.

Optimized parameter \( \delta \) to add some elements to an incomplete HMPR and obtained an incomplete normalized HMPR defined as follows.

Definition 8 (Incomplete N-HMPR) The incomplete HMPR is called incomplete N-HMPR if the known elements are satisfying equation 3.

Zhang and Wu (2014) mentioned that if \( B = (b_{eij})_{n \times n} \) where \( \{b^{\sigma(s)}_{eij}| s = 1, 2, ..., l_{b_{eij}}\} \) be HMPR and \( \bar{B} = (\bar{b}_{eij})_{n \times n} \) be its N-HMPR with the optimized parameter \( \delta \), then \( l \) number of MPR \( p^{(s)}_{ij} = (p^{(s)}_{ij})_{n \times n} \), \( (s = 1, 2, ..., l) \) are obtained where,

\[
p^{(s)}_{ij} = \begin{cases} 
\bar{b}^{\sigma(s)}_{eij} & i < j \\
1 & i = j \\
\bar{b}^{\sigma(l-s+1)}_{eij} & i > j
\end{cases}
\]

In 2014 Zhang and Wu defined the consistent HMPR.

Definition 9 (Zhang and Wu 2014) Let \( B = (b_{eij})_{n \times n} \) be a HMPR and \( \bar{B} = (\bar{b}_{eij})_{n \times n} \) be its N-HMPR with optimized operator \( \delta \). If HMPR is said to be consistent if all the MPRs obtained from \( \bar{B} \) using 4 are consistent.

Definition 10 (Zhang and Wu 2014) Let \( B = (b_{eij})_{n \times n} \) be a HMPR and \( \bar{B} = (\bar{b}_{eij})_{n \times n} \) be its N-HMPR with optimized operator \( \delta \). If HMPR is acceptably consistent if all the MPRs obtained from \( \bar{B} \) using 4 are acceptably consistent.

3 Constructing complete HMPR from an incomplete HMPR

In this section, before discussing constructing complete HMPR from an incomplete HMPR, especially, for an HMPR, \( B = (b_{eij})_{n \times n} \) we have defined the consistency property.
Definition 11 Let $B = (b_{ij})_{n \times n}$ and $\bar{B} = (\bar{b}_{ij})_{n \times n}$ be HMPR and its N-HMPR on X with an optimization operator $\delta$, satisfying the following multiplicative transitive property

$$\bar{b}_{ij}^{(s)} = \bar{b}_{ik}^{\sigma(\bar{b}_{ik} - s + 1)} \times \bar{b}_{kj}^{\sigma(\bar{b}_{kj} - s + 1)}$$

for all $i, j, k \in N$, and $i \neq k \neq j$, $s = 1, 2, \ldots, l_{\bar{b}_{ij}}$.

Definition 12 The incomplete HMPR $B_R = (b_{ij})_{n \times n}$ is called multiplicative consistent if all the known elements satisfy the conditions of equation 5.

Zhang and Wu (2014) proposed the definition of acceptable incomplete HMPR.

Definition 13 (Zhang and Wu 2014) Incomplete HMPR is said to be acceptable incomplete HMPR if at least one known element present in each row or each column of HMPR.

Before going to the next section we have defined an algorithm for constructing a complete HMPR from an incomplete one.

Algorithm 1:
1. Consider an incomplete HMPR, $B_R = (b_{ij})_{n \times n}$
2. Normalized the incomplete HMPR using definition 6 is denoted by $\bar{B}_R = \bar{b}_{ij}$.

$$\bar{b}_{ij} = \begin{cases} * & \text{if } \bar{b}_{ij} \text{ not in } \Omega \\ \bar{b}_{ij} & \text{if } \bar{b}_{ij} \in \Omega \end{cases}$$

Where $\Omega$ is the set of known element and ‘*’ is the value of the missing element.

3. Initially, the missing element can be calculated using the formula

$$\bar{b}_{ij}^{\sigma(\bar{b}_{ij} - s + 1)} = \left( \prod_{k \in T_{ij}} \left( \bar{b}_{ik}^{\sigma(\bar{b}_{ik} - s + 1)} \times \bar{b}_{kj}^{\sigma(\bar{b}_{kj} - s + 1)} \right)^{\frac{1}{t_{ij}}} \right)$$

Where $t_{ij}$ is the number of the elements of the set

$$T_{ij} = \{ k | \bar{b}_{ik}^{\sigma(\bar{b}_{ik} - s + 1)}, \bar{b}_{kj}^{\sigma(\bar{b}_{kj} - s + 1)} \in \Omega \}$$

which indicates that there exist different pairs of adjoining known elements to calculate the missing element $\bar{b}_{ij}$. It is noted that, the subsequent missing elements are evaluated by using the previous calculated missing values.
4. The initial values are obtained. Sometimes initial values obtained may not lie in between $1/9$ to 9 or does not satisfy the required condition of hesitancy as well as consistency. For solving this type of difficulties, we have developed an optimization model that minimizes the error. MATLAB optimization toolbox is used to solve the optimization model. The multiplicative transitive property (Equation 5) of HMPR can be rewritten as

$$\log_{\bar{b}_{e_{ij}}} \sigma (\bar{t}_{e_{ik}} - s + 1) = \log_{\bar{b}_{e_{ik}}} \sigma (\bar{t}_{e_{kj}} - s + 1)$$

$$\forall i, j, k \in N, \text{and } i \neq k \neq j, s = 1, 2, ..., l_{e_{ij}}$$

(B)

In more general inconsistent cases, we can utilize equation 8 to minimize the error. To do this we have constructed the following optimization model.

MODEL (M)

$$\text{Min } \sum_{(i,j)} \mathcal{E}_{e_{ij}}$$

subject to

$$\mathcal{E}_{e_{ij}} = \left| \log_{\bar{b}_{e_{ij}}} \sigma (\bar{t}_{e_{ij}}) - \left( \log_{\bar{b}_{e_{ik}}} \sigma (\bar{t}_{e_{ik}} - s + 1) + \log_{\bar{b}_{e_{kj}}} \sigma (\bar{t}_{e_{kj}} - s + 1) \right) \right| = 0$$

$$\frac{1}{9} - \bar{b}_{e_{ij}} \sigma (s) \leq 0, s = 1, 2, ..., l_{e_{ij}}$$

$$9 - \bar{b}_{e_{ij}} \sigma (s) \leq 0, s = 1, 2, ..., l_{e_{ij}}$$

$$\bar{b}_{e_{ij}} \sigma (s) = \bar{b}_{e_{ij}} \sigma (s) (\ast (0)) \times \bar{b}_{e_{ij}}$$

$$\bar{b}_{e_{ij}} \sigma (s) (\ast (0)) = 1$$

where $\bar{b}_{e_{ij}} \sigma (s)$ are the initial value obtained from the algorithm 1.

Let us consider an example to illustrates the above procedure discuss in section 3.

Example 1 Consider a decision-making problem with five sets of alternatives $x_i = \{x_1, x_2, x_3, x_4, x_5\}$. The decision maker judge these five alternatives by pairwise comparison and provides his/her judgment given in an matrix denoted by $B_R$ given below

$$B_R = \begin{bmatrix}
1,1,1 & 7,5,1 & 3,5,7 & \ast & 3,4,4 \\
3,6,4 & 1,1,1 & 4,6,9 & \ast & 16,24,18 \\
5,6,3 & 1,1,1 & 1,1,1 & \ast & \ast \\
7,6,3 & 10,25,35 & 24,25,16 & \ast & \ast \\
4,7,3 & 1,1,1 & 1,1,1 & \ast & \ast
\end{bmatrix}$$

The missing element can be calculated using equation 7 are given in Table 1.

<table>
<thead>
<tr>
<th>Missing element</th>
<th>Initial value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{e_{14}}$</td>
<td>${16, 16, 12}$</td>
</tr>
<tr>
<td></td>
<td>${5, 5, 5}$</td>
</tr>
<tr>
<td>$b_{e_{24}}$</td>
<td>${3.142, 3.844, 4.189}$</td>
</tr>
<tr>
<td>$b_{e_{34}}$</td>
<td>${0.9153, 0.64, 0.399}$</td>
</tr>
<tr>
<td>$b_{e_{35}}$</td>
<td>${0.2287, 0.16, 0.09986}$</td>
</tr>
</tbody>
</table>
It is to note that the initial value of $\tilde{b}_{e35}$ not lies in between $1/9$ – $9$ ratio scale, therefore error occurs. Thus the error is minimized using minimize the error, the above optimization model (M) solved by MATLAB optimization toolbox. The complete NH-MPR is given by

$$
\begin{bmatrix}
\{1,1,1\} & \{7,5,1\} & \{3,5,7\} & \{16,16,12\} & \{3,4\} \\
\{6,4\} & \{4,6,9\} & \{3.142,3.84,4.189\} & \{16,24,18\} & \{35,25,10\} \\
\{1,1,1\} & \{1,1,1\} & \{0,9153,0,64,0,399\} & \{0,229,0,16,0,111\} \\
\{5,5\} & \{1,1\} & \{1,1\} & \{1,1\} & \{1,1\} \\
\{16,16,12\} & \{1,1\} & \{0,399,0,64,0,915\} & \{0,111,0,16,0,229\} & \{4,4,4\} \\
\{5,5\} & \{1,1\} & \{1,1\} & \{1,1\} & \{1,1,1\} \\
\{4,3\} & \{1,1\} & \{1,1\} & \{1,1\} & \{1,1,1\} \\
\end{bmatrix}
$$

4. Alternative Method for Constructing complete HMPR from an incomplete HMPR:

In this section, we have developed a deviation model by using the consistency property of HMPR to evaluate the missing values.

MODEL(P)

Min $D = \sum_{i,j=1}^{n} \sum_{k \in \Omega(q)} \sum_{s=1}^{l}$ $(d_{ij,k}^{(s)+} + d_{ij,k}^{(s)-})$

Subject to:

$$
\delta_{ij} \left( \log b_{ij}^{(s)} - \left( \log b_{ik}^{(l-s+1)} + \log b_{kj}^{(l-s+1)} \right) \right) - d_{ij,k}^{(s)+} + d_{ij,k}^{(s)-} = 0
$$

$$
\frac{1}{9} \leq b_{ij}^{(s)} \leq 9
$$

$$
b_{ij}^{(s)} \times b_{ij}^{(l-s+1)} = 1
$$

Where

$$
d_{ij,k}^{(s)+} = \left( \log b_{ij}^{(s)} - \left( \log b_{ik}^{(l-s+1)} + \log b_{kj}^{(l-s+1)} \right) \right) \vee 0
$$

$$
d_{ij,k}^{(s)-} = \left( \log b_{ik}^{(l-s+1)} + \log b_{kj}^{(l-s+1)} - \log b_{ij}^{(s)} \right) \vee 0
$$

For easy understanding "l" is the number of element in the each cell of HMPR.

Example 2: Consider a decision-making problem with five sets of alternatives $x_i = \{x_1, x_2, x_3, x_4\}$. The decision maker judge these four alternatives by pair wise comparison and provides his/her judgment given in an matrix denoted by $A$ given below

$$
A = \begin{bmatrix}
\{1\} & \{1,1,1\} & \{1,1\} & \{7,5,1\} \\
\{6,5,4\} & \{3,3,2\} & \{4,6,3\} & \{4,6,3\} \\
\{2,3,3\} & \{1\} & \{1\} & \{1\} \\
\{3,5,7\} & \{1\} & \{1\} & \{1\} \\
\end{bmatrix}
$$

The missing elements are calculated by using Model (P) given in Table 2.
Table 2:

<table>
<thead>
<tr>
<th>Missing element</th>
<th>Calculated value using model(P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{23} \sigma(s)$</td>
<td>${3, 1.667, 1.33}$</td>
</tr>
<tr>
<td>$a_{24} \sigma(s)$</td>
<td>${2, 4.1667, 7}$</td>
</tr>
<tr>
<td>$a_{34} \sigma(s)$</td>
<td>${1, 2.5, 3.5}$</td>
</tr>
</tbody>
</table>

By using 4 the HMPR is split into 3 MPRs.

By using Model (p) the missing values of all three MPRs are calculated and given in table 3:

Table 3

<table>
<thead>
<tr>
<th>MPR</th>
<th>Missing element</th>
<th>Calculated Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A^{(1)}$</td>
<td>$a_{23} \sigma(1)$</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>$a_{24} \sigma(1)$</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>$a_{34} \sigma(1)$</td>
<td>1</td>
</tr>
<tr>
<td>$A^{(2)}$</td>
<td>$a_{23} \sigma(2)$</td>
<td>1.667</td>
</tr>
<tr>
<td></td>
<td>$a_{24} \sigma(2)$</td>
<td>4.1667</td>
</tr>
<tr>
<td></td>
<td>$a_{34} \sigma(2)$</td>
<td>2.5</td>
</tr>
<tr>
<td>$A^{(3)}$</td>
<td>$a_{23} \sigma(3)$</td>
<td>1.33</td>
</tr>
<tr>
<td></td>
<td>$a_{24} \sigma(3)$</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>$a_{34} \sigma(3)$</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Zhu and Xu (2014) gave a method to calculate the maximum satisfaction degree of HMPRs that can deal with hesitant judgment and produce a solution of priorities of the objectives of the decision makers. The satisfaction degree of complete HMPR of example 1 is 0.8024 i.e 80.24% and the weights vectors are $(0.2903, 0.2787, 0.0529, 0.11365, 0.2645)$.

5 Conclusion

In this paper, we have defined a new multiplicative transitive property of HMPR. We have developed an algorithm to construct a complete HMPRs from an incomplete one. Satisfaction degree and acceptably consistent of complete HMPR is also checked. We have presented examples to illustrate the above said methods.

References