ON PRIME VERTEX LABELING OF SOME GRAPHS

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ABSTRACT *A Graph G with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positiveintegers not exceeding n such that the labels of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph. In this paper we investigate the existence of prime labeling of some graphs related to wheel W_n and crowngraphC_n^*.We discuss prime labeling in the context of the graph operation namelyring sum of$P_2 * S_m$.*

Keywords: *Graph Labeling, Prime Labeling,ring sum, Prime Graph.*

1. INTRODUCTION

In this paper, we consider only finite simple undirected graph. The graph G is a set of vertices$V(G)$, together with a set of edges $E(G)$ and incidence relation. If $u, v \in V(G)$are connected by an edge, we say $u\ and\ v$ are adjacent and the corresponding edge is denoted by uvor vu. The degree of a vertex u is the number of edges adjacent with u. A graph is connected if it does not consist of two or more disjoint "pieces". The path P_n is the connected graph consisting of two vertices of degree 1 and $n-2$ vertices of degree 2. An $n-$cycle C_n , is the connected graph consisting of n vertices each of degree 2. An $n-$star S_n, is the graph consisting of one vertex of degree n and n vertices of degree 1. But S_n consist of $n+1$ vertices and n edges. A tree is a graph contains no cycle. Path and stars are example of trees.For notations and terminology we refer to Bondy and Murthy [1].

The graph obtained by attaching a pendant edge at every vertex of G is denoted by G^*

A complete binary tree is a directed rooted tree with every internal vertex having two children. In [3], Seoud and Youssef showed that every cycle with identical complete binary trees attached to each cycle vertex is co-prime.

A complete binary tree is a directed rooted tree with every internal vertex having three children.A one – level complete ternary tree is equal to the star S_3 .

We define an n - cycle –pendant with 1- level ternary tree , denoted $C_n * P_2 * S_3$, to be the graph that results from first attaching a single pendant to each cycle vertex of C_n followed by attaching a one – level complete ternary tree to each pendant vertex . The star notation means that a cycle C_n , with n vertices attach one more vertex to each cycle vertex, C_i. Now, we have each C_i as one side of a path and one vertex of degree one , denoted P_i attach three more vertices of degree one to each P_i , making each P_i the center vertex of a star , S_3 [4].

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout [5]. Two integers a and b are said to be relatively prime if their greatest common divisor is 1 denoted by $(a,b)=1$. Every path P_n, cycle C_n and star S_n are prime [2] . Every wheel W_n iff n is even [2], all helm H_n , crown C_n^* and gear graph G_n are prime [2]. We refer Gallian's dynamic survey [2] for a comprehensive listing of the families of graphs that are known to have or known not to have prime vertex labeling.

The crown graph C_n^* is obtained from a cycle $C_n$by attaching a pendent edge at each vertex of the n-cycle.

The graph obtained by, attaching the center vertex of a copy of the star S_m to the path P_2. The resulting graph will have m- pendants at one vertex of $P_2$and it is denoted by $P_2 * S_m$.

The graph obtained by, attaching the copy of $P_2 * S_m$ to each vertex of Wheel W_n and crown C_n^* . The resulting graph is denoted by $W_n * P_2 * S_m$and $C_n^* * P_2 * S_m$.

In this paper, we proved that the graphs obtained by attaching the copy of $P_2 * S_m$ to each vertex of Wheel W_n and crown C_n^* are all prime graphs.

2. MAIN RESULTS

Theorem 2.1

The graph $W_n * P_2 * S_3$ admits prime labeling where W_n is the wheel graph. if $n \not\equiv 1(mod\ 3)$

Proof.

Let c_0 be the centre of the wheel graph W_n. And let $c_1, c_2, \dots c_n$ be the rim vertices of the wheel graph W_n and let $p_0, p_1, p_2, \dots p_n$ be the pendant vertices adjacent to $c_0, c_1, c_2, \dots c_n$ respectively. letp_i^j, be the pendant vertices attached at eachp_i, $1 \le i \le n$ and $1 \le j \le 3$respectively. And let p_0^j , $1 \le j \le 3$ be the pendant vertices attached at p_0.

The graph $W_n * P_2 * S_3$ has $5n + 5$ vertices and $6n + 4$ edges.

Define a labeling $f : V \to \{1,2,3, \dots, 5n + 5\}$as follows.

Let $f(c_0) = 1, f(P_0) = 5$,

$f(p_0^j) = j + 1,\ \ for\ j = 1,3\ \ , f(c_1) = 3, f(p_0^2) = 6.$

For $\boldsymbol{1 \le i \le n}$

$$f(c_i) = 5i + 1,$$
$$f(p_i) = 5i + 2, if\ i\ is\ oddand\ \ for\ i + 1 \not\equiv 0(mod\ 6)$$
$$f(p_i) = 5i + 4, if\ i\ is\ oddand\ \ i + 1 \equiv 0(mod\ 6)$$
$$f(p_i) = 5i + 3, if\ i\ is\ even$$
$$f(p_i^1) = 5i + 3\ , if\ i\ is\ odd$$
$$f(p_i^1) = 5i + 2\ , if\ i\ is\ even$$
$$f(p_i^2) = 5i + 4\ ,\quad if\ i\ is\ oddand\ \ i + 1 \not\equiv 0(mod\ 6)\ ,$$
$$f(p_i^2) = 5i + 2\ ,\quad if\ i\ is\ oddand\ \ i + 1 \equiv 0(mod\ 6)\ ,$$

$f(p_i^3) = 5i + 5$

$\gcd(f(c_0), f(p_0)) = \gcd(1,5\ ) = 1.$

$\gcd(f(c_0), f(c_1)) = \gcd(1,3) = 1$

$\gcd(f(c_0), f(c_i)) = \gcd(1, 5i + 1) = 1, for\ 2 \le i \le n$

$\gcd(f(p_0^1), f(p_0),) = \gcd(2,5) = 1$

$\gcd(f(p_0^2), f(p_0),) = \gcd(6,5) = 1$

$\gcd(f(p_0^3), f(p_0),) = \gcd(4,5) = 1$

$\gcd(f(c_1), f(c_2))\ = \gcd(3,11) = 1$

$\gcd(f(c_1), f(p_1))\ = \gcd(3,7\ ) = 1.$

$\gcd(f(c_i), f(c_{i+1})) = \gcd(5i + 1, 5(i + 1) + 1)$

$\qquad = \gcd(5i + 1, (5i + 1) + 5) = 1\ , for\ 2 \le i \le n$

among these two numbers one is odd and other is even their difference is 5 and they are not multiples of 5.

$\gcd(f(c_1), f(c_n)) = \gcd(3, 5n + 1) = 1,\quad since\ n \not\equiv 1(mod\ 3)$

If i is odd and $\boldsymbol{i + 1 \not\equiv 0(mod\ 6)}$

$$\gcd(f(c_i), f(p_i)) = \gcd(5i + 1, 5i + 2\ ) = 1$$

$\gcd(f(p_i), f(p_i^1)) = \gcd(5i + 2, 5i + 3) = 1$

as these two number are consecutive integers.

$\gcd(f(p_i), f(p_i^2)) = \gcd(5i + 2, 5i + 4) = 1$

as these two number are consecutive odd integers.

$\gcd\ (f(p_i), f(p_i^3)) = \gcd(5i + 2, 5i + 5) = 1$

among these two numbers one is odd and other is even their difference is 3 and they are not multiples of 3.

If i is odd and $\boldsymbol{i + 1 \equiv 0(mod\ 6)}$

$$\gcd(f(c_i), f(p_i)) = \gcd(5i + 1, 5i + 4\ ) = 1$$

among these two numbers one is odd and other is even their difference is 3 and they are not multiples of 3.

$\gcd(f(p_i), f(p_i^1)) = \gcd(5i + 4, 5i + 3) = 1$

$\gcd\ (f(p_i), f(p_i^3)) = \gcd(5i + 4, 5i + 5) = 1$

as these two number are consecutive integers.

$\gcd(f(p_i), f(p_i^2)) = \gcd(5i + 4, 5i + 2) = 1$

as these two number are consecutive odd integers.

$$\boldsymbol{If \quad i\, is\, even}$$
$$\gcd\big(f(c_i), f(p_i)\big) = \gcd(5i+1, 5i+3) = 1$$

$\gcd\left(f(p_i), f(p_i^3)\right) = \gcd(5i+3, 5i+5) = 1$

as these two number are consecutive odd integers.

$\gcd\left(f(p_i), f(p_i^1)\right) = \gcd(5i+3, 5i+2) = 1$

$\gcd\left(f(p_i), f(p_i^2)\right) = \gcd(5i+3, 5i+4) = 1$

as these two number are consecutive integers.

Thus f is a prime labeling.

Hence $W_n * P_2 * S_3$ is a prime graph.

Theorem 2.2

The graph $W_n * P_2 * S_5$ admits prime labeling where W_n is the wheel graph.if$n \not\equiv 1(mod\ 3)$

Proof.

Let c_0 be the centre of the wheel graph W_n. And let $c_1, c_2, \dots c_n$ be the rim vertices of the wheel graph W_n and let $p_0, p_1, p_2, \dots, p_n$ be the pendant vertices adjacent to $c_0, c_1, c_2, \dots, c_n$ respectively. letp_i^j be the pendant vertices attached at each p_i, $1 \le i \le n$ and $1 \le j \le 5$. And let p_0^j , $1 \le j \le 5$,be the pendant vertices attached at p_0.

The graph $W_n * P_2 * S_5$ has $7n+7$ vertices and $8n+6$ edges.

Define a labeling $f : V \to \{1,2,3, \dots, 7n+7\}$as follows.

Let$f(c_0) = 1, f(P_0) = 7$,

$f(p_0^j) = j+1$, $for\ 1 \le j \le 5, f(c_1) = 3, f(p_1^4) = 8$.

$\boldsymbol{For\ 1 \le i \le n}$

$$f(c_i) = 7i+1,$$
$$\boldsymbol{If\ i\ is\ odd}$$
$$f(p_i) = 7i+4, for\ \ i+1 \not\equiv 0(mod\ 6)$$
$$f(p_i) = 7i+6, for\ \ i+1 \equiv 0(mod\ 6)\ \ and\, i+13 \not\equiv 0(mod\ 30)$$
$$f(p_i) = 7i+2, fori+13 \equiv 0(mod\ 30)$$
$$f(p_i^1) = 7i+4\ , for\ \ i+13 \equiv 0(mod\ 30)$$
$$f(p_i^1) = 7i+2\ , for\ \ i+13 \not\equiv 0(mod\ 30)$$
$$f(p_i^2) = 7i+3\ ,\ \ f(p_i^3) = 7i+5$$
$$f(p_i^4) = 7i+6\ , for\ \ i+1 \not\equiv 0(mod\ 6)\ \ and\ \ i+13 \equiv 0(mod\ 30)$$
$$f(p_i^4) = 7i+4\ \ , for\ \ i+1 \equiv 0(mod\ 6)\ \ and\, i+13 \not\equiv 0(mod\ 30)$$

$f(p_i^5) = 7i+7$,

$$\boldsymbol{If \quad i\, is\, even}$$
$$f(p_i) = 7i+3, for\ \ i \not\equiv 0(mod\ 6)$$
$$f(p_i) = 7i+7, for\ \ i \equiv 0(mod\ 6)\ \ and\, i+6 \not\equiv 0(mod\ 30)$$
$$f(p_i) = 7i+5, for\ \ i+6 \equiv 0(mod\ 30)$$
$$f(p_i^1) = 7i+2\ , f(p_i^2) = 7i+4\ ,$$

$f(p_i^3) = 7i+5,\ \ for\ \ i+6 \not\equiv 0(mod\ 30)$

$f(p_i^3) = 7i+3,\ \ for\ \ i+6 \equiv 0(mod\ 30)$

$f(p_i^4) = 7i+6$

$$f(p_i^5) = 7i+7\ , for\, i \not\equiv 0(mod\ 6)\ \ and\ \ i+6 \equiv 0(mod\ 30)$$
$$f(p_i^5) = 7i+3\ \ , for\, i \equiv 0(mod\ 6)\ \ and\ \ i+6 \not\equiv 0(mod\ 30)$$

Similar to previous theorem we can prove that $\gcd\big(f(u), f(v)\big) = 1$ for all edges uv in G.

Thus f is a prime labeling.

Hence $W_n * P_2 * S_5$ is a prime graph.

Illustration 2.1

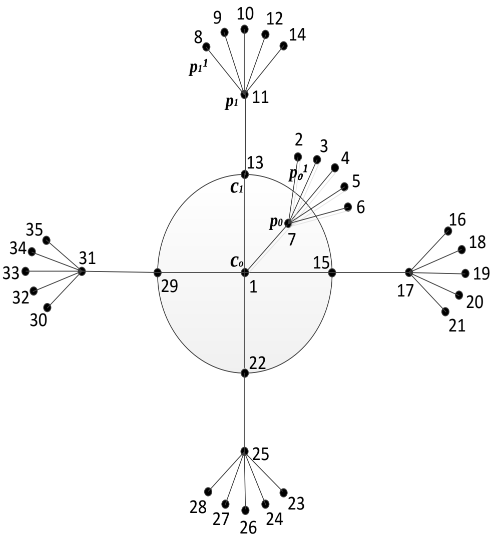

Figure 1. Prime labeling for $W_n * P_2 * S_5$

Theorem 2.3
The graph $W_n * P_2 * S_7$ admits prime labeling .if $n + 3 \not\equiv 0(mod\ 13)$
Proof.

Let c_0 be the centre of the wheel graph W_n. And let $c_1, c_2, \dots c_n$ be the rim vertices of the wheel graph W_n and let $p_0, p_1, p_2, \dots, p_n$ be the pendant vertices adjacent to $c_0, c_1, c_2, \dots, c_n$ respectively. letp_i^j be the pendant vertices attached at each p_i, $1 \le i \le n$ and $1 \le j \le 7$. And let p_0^j , $1 \le j \le 7$,be the pendant vertices attached at p_0.

The graph $W_n * P_2 * S_7$ has $9n + 9$ vertices and $10n + 8$ edges.

Define a labeling $f : V \to \{1,2,3, \dots, 9n + 9\}$as follows.

Let$f(c_0) = 1, f(P_0) = 7$,

$f(p_0^j) = j + 1,\ \ for\ 1 \le j \le 7, f(c_1) = 13, f(p_1^2) = 10$.

***For* $1 \le i \le n$**

$$f(c_i) = 9i + 1,$$

If i is odd

$$f(p_i) = 9i + 2, for\ \ i + 3 \not\equiv 0(mod\ 10), i + 15 \not\equiv 0(mod\ 28)\ and\ \ i + 71 \not\equiv 0(mod\ 140)$$

$f(p_i) = 9i + 4, for\ \ i + 3 \equiv 0(mod\ 10),\ \ i + 15 \equiv 0(mod\ 28)\ andi + 71 \not\equiv 0(mod\ 140)$

$$f(p_i) = 9i + 8, for\ \ i + 71 \equiv 0(mod\ 140)$$

$$f(p_i^1) = 9i + 3,$$

$f(p_i^2) = 9i + 4, for\ \ i + 13 \not\equiv 0(mod\ 10), i + 15 \not\equiv 0(mod\ 28) and\ \ i + 71 \equiv 0(mod\ 140)$

$f(p_i^2) = 9i + 2, for\ \ i + 13 \equiv 0(mod\ 10), i + 15 \equiv 0(mod\ 28)\ and\ \ i + 71 \not\equiv 0(mod\ 140)$

$f(p_i^3) = 9i + 5, f(p_i^4) = 9i + 6\ \ , f(p_i^5) = 9i + 7,$

$f(p_i^6) = 9i + 8, for\ \ i + 71 \not\equiv 0(mod\ 140)$

$f(p_i^6) = 9i + 2, for\ \ i + 71 \equiv 0(mod\ 140)$

$f(p_i^7) = 9i + 9.$

If i is even

$f(p_i) = 9i + 5, f(p_i^1) = 9i + 2\ , f(p_i^2) = 9i + 3\ ,\ f(p_i^3) = 9i + 4,\ f(p_i^4) = 9i + 6,$

$$f(p_i^5) = 9i + 7, f(p_i^6) = 9i + 8\ , f(p_i^7) = 9i + 9.$$

For any edge $e = uv$ of G we can prove that $\gcd(f(u), f(v)) = 1$.

Thus f is a prime labeling.

Hence $W_n * P_2 * S_7$ is a prime graph.

Illustration 2.2

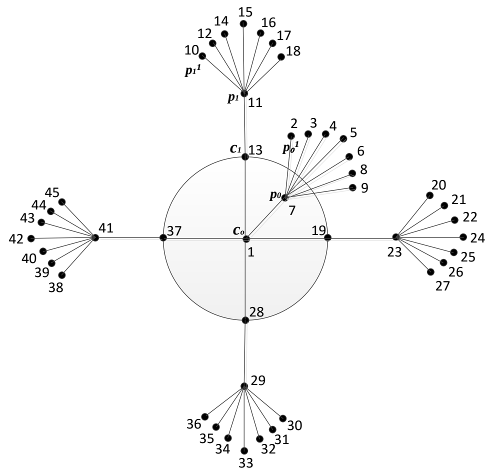

Figure 2. Prime labeling for $W_n * P_2 * S_7$

Theorem 2.4

The graph $C_n^* * P_2 * S_3$ admits prime labeling where C_n^* is the crown graph .

Proof.

Let $c_1, c_2, \ldots c_n$ be the rim vertices of the crown graph C_n^* . and let $p_1, p_2, \ldots p_n$ be the pendant vertices attached at $c_1, c_2, \ldots c_n$ respectively. Let $s_1, s_2, \ldots, s_n$ be the pendant vertices attached at $p_1, p_2, \ldots, p_n$ and let p_i^j, $1 \le i \le n, 1 \le j \le 3$, be the pendant vertices attached at each s_i , $1 \le i \le n$.

The graph $C_n^* * P_2 * S_3$ has $6n$ vertices and $6n$ edges.

Define a labeling $f : V \to \{1,2,3, \ldots, 6n\}$ as follows.

***For* $1 \le i \le n$**

$$f(c_i) = 6i - 5, \qquad f(p_i) = 6i - 4, f(s_i) = 6i - 1,$$
$$f(p_i^1) = 6i - 3 \, , f(p_i^2) = 6i - 2 \, , f(p_i^3) = 6i$$

***For* $1 \le i \le n$**

$\gcd(f(c_i), f(c_{i+1})) = \gcd(6i - 5, 6(i + 1) - 5)$

$= \gcd(6i - 5, (6i - 5) + 6) = 1$,

as these two numbers are odd and their difference is 6 and both are not multiples of 3 or 5.

$\gcd(f(c_1), f(c_n)) = \gcd(1, \; 6n - 5) = 1,$

$$\gcd(f(c_i), f(p_i)) = \gcd(6i - 5, 6i - 4) = 1,$$

as these two number are consecutive integers.

$\gcd(f(p_i), f(s_i)) = \gcd(6i - 4, 6i - 1) = 1,$

among these two numbers one is even and other is odd and their difference is 3and they are not multiples of 3.

$\gcd\left(f(p_i^1), f(s_i)\right) = \gcd(6i - 3, 6i - 1) = 1,$

as these two number are consecutive odd integers.

$\gcd\left(f(p_i^2), f(s_i)\right) = \gcd(6i - 2, 6i - 1) = 1,$

$\gcd\left(f(p_i^3), f(s_i)\right) = \gcd(6i, 6i - 1) \qquad = 1,$

as these two number are consecutive integers.

Thus f is a prime labeling.

Hence $C_n^* * P_2 * S_3$ is prime graph.

Theorem 2.5

The graph $C_n^* * P_2 * S_5$ admits prime labeling where C_n^* is the crown graph .

Proof.

Let $c_1, c_2, \ldots c_n$ be the rim vertices of the crown graph C_n^* . and let $p_1, p_2, \ldots p_n$ be the pendant vertices attached at $c_1, c_2, \ldots c_n$ respectively. Let $s_1, s_2, \ldots, s_n$ be the pendant vertices attached at each $p_1, p_2, \ldots, p_n$ and let p_i^j, $1 \le i \le n, 1 \le j \le 5$, be the pendant vertices attached at each s_i , $1 \le i \le n$.

The graph $C_n^* * P_2 * S_5$ has $8n$ vertices and $8n$ edges.
Define a labeling $f : V \to \{1,2,3,\dots,8n\}$ as follows.
For $\boldsymbol{1 \le i \le n}$

$$f(c_i) = 8i - 7, \quad f(p_i) = 8i - 6,$$
$$f(s_i) = 8i - 3, for\ \ i \not\equiv 0(mod\ 3)$$
$$f(s_i) = 8i - 5, for\ \ i \equiv 0(mod\ 3), \quad i \not\equiv 0(mod\ 15)$$
$$f(s_i) = 8i - 1, for\ \ i \equiv 0(mod\ 15),$$
$$f(p_i^1) = 8i - 5\ , for\ \ i \not\equiv 0(mod\ 3), \quad i \equiv 0(mod\ 15)$$
$$f(p_i^1) = 8i - 3\ , for\ \ i \equiv 0(mod\ 3), \quad i \not\equiv 0(mod\ 15)$$
$$f(p_i^2) = 8i - 4\ , f(p_i^3) = 8i - 2,$$

$f(p_i^4) = 8i - 1, for\ \ i \not\equiv 0(mod\ 15),$
$f(p_i^4) = 8i - 3, for\ \ i \equiv 0(mod\ 15)\ ,$
$f(p_i^5) = 8i$
Similar to previous theorem we can prove that $\gcd(f(u), f(v)) = 1$ for all edges uv in G.
Thus f is a prime labeling.
Hence $C_n^* * P_2 * S_5$ is a prime graph.

Illustration 2.3

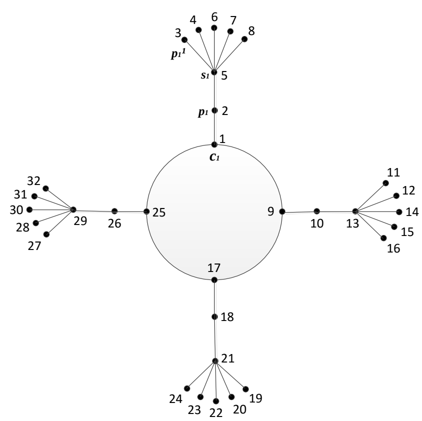

Figure 3. Prime labeling for $\boldsymbol{C_n^* * P_2 * S_5}$

Theorem 2.6
The graph $C_n^* * P_2 * S_7$ admits prime labeling where C_n^* is the crown graph .
Proof.

Let $c_1, c_2, \dots c_n$ be the rim vertices of the crown graph C_n^* . and let $p_1, p_2, \dots p_n$ be the pendant vertices attached at $c_1, c_2, \dots c_n$ respectively. Let $s_1, s_2, \dots, s_n$ be the pendant vertices attached at each $p_1, p_2, \dots, p_n$ and let p_i^j, $1 \le i \le n$, $1 \le j \le 7$, be the pendant vertices attached at each s_i , $1 \le i \le n$.
The graph $C_n^* * P_2 * S_7$ has $10n$ vertices and $10n$ edges.
Define a labeling $f : V \to \{1,2,3,\dots,10n\}$ as follows.
For $\boldsymbol{1 \le i \le n}$

$$f(c_i) = 10i - 9, \quad f(p_i) = 10i - 8,$$
$$f(s_i) = 10i - 3, for\ i \not\equiv 0(mod\ 3)$$
$$f(s_i) = 10i - 7, for\ i \equiv 0(mod\ 3), \quad i \not\equiv 1(mod\ 20)$$
$$f(s_i) = 10i - 1, for\ i \equiv 1(mod\ 20)$$
$$f(p_i^1) = 10i - 7\ , for\ i \not\equiv 0(mod\ 3), \quad i \equiv 1(mod\ 20)$$
$$f(p_i^1) = 10i - 3\ , for\ i \equiv 0(mod\ 3), \quad i \not\equiv 1(mod\ 20)$$

$f(p_i^2) = 10i - 6\ , f(p_i^3) = 10i - 5,\ f(p_i^4) = 10i - 4, f(p_i^5) = 10i - 2,$
$f(p_i^6) = 10i - 1,\ for\ i \not\equiv 1(mod\ 20)$
$f(p_i^6) = 10i - 3, for\ i \equiv 1(mod\ 20)$
$f(p_i^7) = 10i.$
For any edge $e = uv$ of G we can prove that $\gcd(f(u), f(v)) = 1$.
Thus f is a prime labeling.
Hence $C_n^* * P_2 * S_7$ is a prime graph.

Illustration 2.4

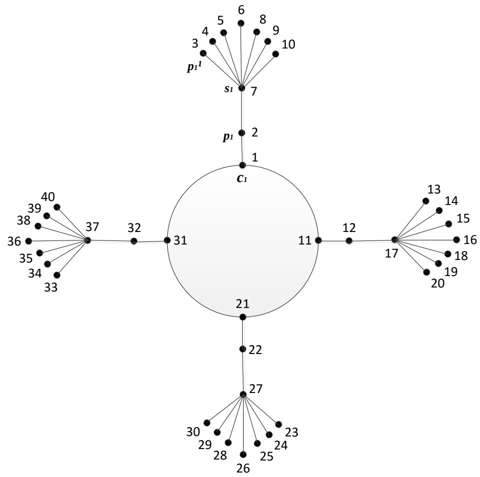

Figure 4. Prime labeling for $C_n^* * P_2 * S_7$

Theorem 2.7

The graph $C_n^* * P_2 * S_9$ admits prime labeling where C_n^* is the crown graph.

Proof.

Let $c_1, c_2, \ldots c_n$ be the rim vertices of the crown graph C_n^* . and let $p_1, p_2, \ldots p_n$ be the pendant vertices attached at $c_1, c_2, \ldots c_n$ respectively. Let $s_1, s_2, \ldots, s_n$ be the pendant vertices attached at each $p_1, p_2, \ldots, p_n$ and let p_i^j, $1 \le i \le n$, $1 \le j \le 9$,be the pendant vertices attached at each s_i , $1 \le i \le n$.

The graph $C_n^* * P_2 * S_9$ has $12n$ vertices and $12n$ edges.

Define a labeling $f : V \to \{1,2,3, \ldots, 12n\}$ as follows.

***For* $1 \le i \le n$**

$$f(c_i) = 12i - 11, \qquad f(p_i) = 12i - 10,$$
$$f(s_i) = 12i - 5, for\ i \not\equiv 0 (mod\ 5)$$
$$f(s_i) = 12i - 7, for\ i \equiv 0 (mod\ 5)\ , \qquad i \not\equiv 0 (mod\ 35)$$
$$f(s_i) = 12i - 1, for\ i \equiv 0 (mod\ 35)$$
$$f(p_i^1) = 12i - 9\ , f(p_i^2) = 12i - 8\ ,$$
$$f(p_i^3) = 12i - 7\ , \qquad for\ i \not\equiv 0 (mod\ 5)$$
$$f(p_i^3) = 12i - 5, \qquad for\ i \equiv 0 (mod\ 5)\ , \qquad i \not\equiv 0 (mod\ 35)$$

$f(p_i^4) = 12i - 6,\ f(p_i^5) = 12i - 4,\ f(p_i^6) = 12i - 3,\ f(p_i^7) = 12i - 2$

$f(p_i^8) = 12i - 1,\ for\ i \not\equiv 0 (mod\ 35)$

$f(p_i^8) = 12i - 5,\ for\ i \equiv 0 (mod\ 35)$

$f(p_i^9) = 12i.$

We can prove that in a similar way that $\gcd(f(u), f(v)) = 1$ for any edges uv of G.

Thus f is a prime labeling.

Hence $C_n^* * P_2 * S_9$ is a prime graph.

Theorem 2.8

The graph $C_n^* * P_2 * S_{11}$ admits prime labeling where C_n^* is the crown graph .

Proof.

Let $c_1, c_2, \ldots c_n$ be the rim vertices of the crown graph C_n^* . and let $p_1, p_2, \ldots p_n$ be the pendant vertices attached at $c_1, c_2, \ldots c_n$ respectively. Let $s_1, s_2, \ldots, s_n$ be the pendant vertices attached at each $p_1, p_2, \ldots, p_n$ and let p_i^j, $1 \le i \le n$, $1 \le j \le 11$,be the pendant vertices attached at each s_i , $1 \le i \le n$.

The graph $C_n^* * P_2 * S_{11}$ has $14n$ vertices and $14n$ edges.

Define a labeling $f : V \to \{1,2,3, \ldots, 14n\}$ as follows.

***For* $1 \le i \le n$**

$$f(c_i) = 14i - 13, \qquad f(p_i) = 14i - 12,$$

$f(s_i) = 14i - 1, for\ \ i \not\equiv_{15} 2,4,5,8,9,11,14\ , i + 18 \not\equiv_{33} 0, \quad i \not\equiv_{33} 4.$

$f(s_i) = 14i - 9, for\ \ i \equiv_{15} 2,4,5,8,14\ , \quad i + 128 \equiv_{165} 0.$

$$f(s_i) = 14i - 5, for\ \ i \equiv_{15} 9,11, i + 18 \equiv_{33} 0, i + 150 \not\equiv_{165} 0.$$
$$f(s_i) = 14i - 11, for\ \ i + 150 \equiv_{165} 0.$$
$$f(s_i) = 14i - 3, for\ \ i \equiv_{33} 4,\ \ i + 128 \not\equiv_{165} 0.$$
$$f(p_i^1) = 14i - 11\ , for\ \ i + 150 \not\equiv_{165} 0\ .$$
$$f(p_i^1) = 14i - 1\ , for\ \ i + 150 \equiv_{165} 0.$$
$$f(p_i^2) = 14i - 10\ ,$$
$$f(p_i^3) = 14i - 9\ ,\ \ \ for\ \ i \not\equiv_{15} 2,4,5,8,14, i + 128 \not\equiv_{165} 0.$$
$$f(p_i^3) = 14i - 1,\ \ \ for\ \ i \equiv_{15} 2,4,5,8,14\ ,\ \ \ i + 128 \equiv_{165} 0.$$

$f(p_i^4) = 14i - 8,\ f\left(p_i^5\right) = 14i - 7,\ f(p_i^6) = 14i - 6,$
$f(p_i^7) = 14i - 5, for\ \ i \not\equiv_{15} 9,11\ , i + 18 \not\equiv_{33} 0, i + 150 \equiv_{165} 0.$
$f(p_i^7) = 14i - 1, for\ \ i \equiv_{15} 9,11\ , i + 18 \equiv_{33} 0.$
$f(p_i^8) = 14i - 4,$
$f(p_i^9) = 14i - 3,\ for\ \ i \not\equiv_{33} 4\ , i + 128 \equiv_{165} 0.$
$f(p_i^9) = 14i - 1, for\ \ i \equiv_{33} 4\ .$
$f(p_i^{10}) = 14i - 2, f(p_i^{11}) = 14i.$
We can prove that in a similar way that $\gcd\big(f(u), f(v)\big) = 1$ for any edges $uv$of G.
Thus f is a prime labeling.
Hence $C_n^* * P_2 * S_{11}$ is a prime graph.

3. CONCLUSION

wehave presented eight new results on the prime labeling certain classes of graphs like attaching the copy of $P_2 * S_m$ to each vertex of Wheel W_n and crown C_n^*. It is very difficult to generalize these labeling due to the nature of the prime numbers.Analogous work can be carried out for other families and in the context of different types of graph labeling techniques.

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