Transportation ProblemUsing Intuitionistic Decagonal Fuzzy Number

B.Anushya[1] & B.Ramaand[2] & L.Sudha[3]
[1]P.G.Scholar,Department of Mathematics,Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore.
[2 3]Assistant Professor, Department of Mathematics,Avinashilingam Institute for Home Science and Higher Education for Women, Coimbatore.

Received: January 26, 2019 Accepted: March 08, 2019

ABSTRACT: *In this research paper a new ranking method is introduced forIntuitionistic decagonal fuzzy numbers. By using this ranking a fuzzy transportation problem is converted to a crisp valued problem, then itis solved using VAM method for initial solution and MODI method for optimal solution. A numerical example is given to show that the new ranking procedure gives optimum transportation cost.*

Key Words: : *Fuzzy Number, Decagonal fuzzy number, Fuzzy Transportation problem*

1 Introduction

Fuzzy set was first developed by L.A.Zadeh[14]. Later its application was developed by H.J.Zimmermann[13].MichaelO'hEigeartaigh[7] determined an algorithm for solving fuzzy transportation problem. A new method for solving fuzzy transportation problem was introduced by Nagoorkani A, Abbas[8]. P.Pandian and G.Natarajan[10] found a new algorithm for finding fuzzy optimal solution. Many researchers used various fuzzy numbers in transportation problem to minimize the transportation cost.

The paper is organized as follows: In Section 2, basic definition of fuzzy number and decagonal fuzzy numbers are given. In Section 3, existing ranking method, new ranking method are defined. In Section 4, a numerical example (FTP) is solved to illustrate the new ranking method and also to compare the result of the existing ranking method. Finally the paper ends with a conclusion.

2 Basic Definitions :

Definition 2.1. A fuzzy set $\breve{A}$ in X is characterized by a membership function $\mu_{\breve{A}}(x)$which associates each point in X, to a real number in the interval [0, 1]. The value of$\mu_{\breve{A}}(x)$ represents "grade of membership" of x $\in\mu_{\breve{A}}(x)$. More general representation for a fuzzy set is $\breve{A}$ = {(x,$\mu_{\breve{A}}(x)$)}.

Definition 2.2. The α−cut of the fuzzy set $\breve{A}$ of the universe of discourse X is defined as $\widetilde{A_\alpha}$ = {x ∈ X/$\mu_{\breve{A}}(x)$) ≥ α}, where α ∈ [0, 1].

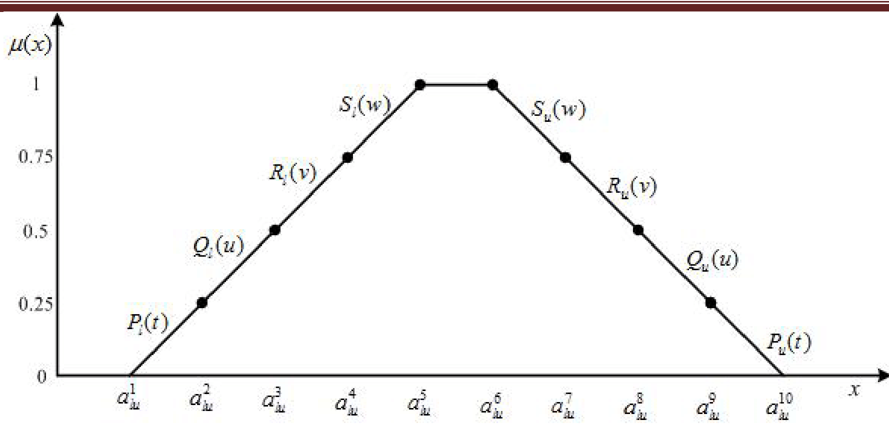
Definition 2.3. A fuzzy set$\breve{A}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function $\breve{A}$: R → [0, 1] has the following characteristics.

(i) $\breve{A}$ is convex $\mu_{\breve{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min(\mu_{\breve{A}}(x_1),\mu_{\breve{A}}(x_2))$,
∀x ∈ [0, 1], λ ∈ [0, 1]

(ii) $\breve{A}$ is normal i.e. there exists an x ∈ R such that if max $\mu_{\breve{A}}(x)$= 1.

(iii) $\breve{A}$is piecewise continuous.

Definition 2.4.A Decagonal fuzzy number $\breve{D}$denoted as (a1, a2, a3, a4, a5, a6, a7, a8, a9, a10), and the membership function andnonmembershipfunction defined as



$$\mu_{\breve{D}}(x)=\begin{cases} 0, x<a_1 \\ \dfrac{1(x-a_1)}{4(a_2-a_1)}, a_1 \le x \le a_2 \\ \dfrac{1}{4}+\dfrac{1(x-a_2)}{4(a_3-a_2)}, a_2 \le x \le a_3 \\ \dfrac{1}{2}+\dfrac{1(x-a_3)}{4(a_4-a_3)}, a_3 \le x \le a_4 \\ \dfrac{3}{4}+\dfrac{1(x-a_4)}{4(a_4-a5)}, a_4 \le x \le a_5 \\ 1 \quad , a_5 \le x \le a_6 \\ 1-\dfrac{1(x-a_6)}{4(a_7-a_6)}, a_6 \le x \le a_7 \\ \dfrac{3}{4}-\dfrac{1(x-a_7)}{4(a_8-a_7)}, a_7 \le x \le a_8 \\ \dfrac{1}{2}-\dfrac{1(x-a_8)}{4(a_9-a_8)}, a_8 \le x \le a_9 \\ \dfrac{1(a_{10}-x)}{4(a_{10}-a_9)} a_9 \le x \le a_{10} \\ 0\ , otherwise \end{cases}$$

$$\gamma_{\breve{D}}(x)=\begin{cases} 1, x<b_1 \\ 1-\dfrac{1(b_2-x)}{4(b_2-b_1)}, b_1 \le x \le b_2 \\ \dfrac{3}{4}-\dfrac{1(b_3-x)}{4(b_3-b_2)}, b_2 \le x \le b_3 \\ \dfrac{1}{2}-\dfrac{1(b_4-x)}{4(b_4-b_3)}, b3 \le x \le b_4 \\ \dfrac{1(b_5-x)}{4(b_5-b_4)}, b_4 \le x \le b_5 \\ 0 \quad , b_5 \le x \le b_6 \\ \dfrac{1(b_7-x)}{4(b_7-b_6)}, b_6 \le x \le b_7 \\ \dfrac{1}{4}+\dfrac{1(b_8-x)}{4(b_8-b_7)}, b_7 \le x \le b_8 \\ \dfrac{1}{2}+\dfrac{1(b_9-x)}{4(b_9-b_8)}, b_8 \le x \le b_9 \\ \dfrac{3}{4}+\dfrac{1(x-b_9)}{4(b_{10}-b_9)} b_9 \le x \le b_{10} \\ 1, otherwise \end{cases}$$

Definition 2.5. The α-cut of the fuzzy set of the universe of discourse X is defined as
$\widetilde{D_\alpha}$ = {x ∈ X / $\mu_{\breve{A}}(x)$ ≥ α}, where α ∈ [0,1].

$$\widetilde{D_\alpha} = \begin{cases} [P_l(\alpha), P_u(\alpha)], for\ \alpha \in [0,0.25) \\ [Q_l(\alpha), Q_u(\alpha)], for\ \alpha \in [0.25,0.5) \\ [R_l(\alpha), R_u(\alpha)], for\ \alpha \in [0.5,0.75) \\ [S_l(\alpha), S_u(\alpha)], for\ \alpha \in [0.75,1] \end{cases}$$

$$\widetilde{D_\alpha} = \begin{cases} [\overline{P_l(\alpha)}, \overline{P_u(\alpha)}], for\ \alpha \in [0,0.25) \\ [\overline{Q_l(\alpha)}, \overline{Q_u(\alpha)}], for\ \alpha \in [0.25,0.5) \\ [\overline{R_l(\alpha)}, \overline{R_l(\alpha)}], for\ \alpha \in [0.5,0.75) \\ [\overline{S_l(\alpha)}, \overline{S_l(\alpha)}], for\ \alpha \in [0.75,1] \end{cases}$$

If $P_l(x)$ = α and $P_u(x)$ = α then α-cut operations interval$\widetilde{D_\alpha}$ is obtained as
(1). $[P_l(\alpha),P_u(\alpha)] = [4\alpha(a_1- a_1) + a_1,-4\alpha(a_{10}- a_9) + a_{10}]$.
Similarly, we can obtain α-cut operations interval Dα for $[Q_l(\alpha),Q_u(\alpha)]$, $[R_l(\alpha),R_u(\alpha)]$ and $[S_l(\alpha),S_u(\alpha)]$ as follows
(2). $[Q_l(\alpha),Q_u(\alpha)] = [4\alpha(a_3 -a_2) + 2a_2 -a_3,-4\alpha(a_9 -a_8) + 2a_9 -a_8]$
(3). $[R_l(\alpha),R_u(\alpha)] = [4\alpha(a_4 -a_3) + 3a_3 -2a_4,-4\alpha(a_8 -a_7) + 3a_8-2a_7]$
(4). $[S_l(\alpha),S_u(\alpha)] = [4\alpha(a_5 -a_4) + 4a_4 -3a_5,-4\alpha(a_7 -a_6) + 4a_7 -3a_6]$
For a non- membership function,
(1).$[\overline{P_l(\alpha)}, \overline{P_u(\alpha)}]$ = [4 α (b2-b1) – 3b2+4b1 , (-4 α(b10-b9)) -3b10+4b9]
(2).$[\overline{Q_l(\alpha)}, \overline{Q_u(\alpha)}]$=[4 α (b3-b2) – 2b3+3b2 , (-4 α(b9-b8)) -2b8+3b9]
(3).$[\overline{R_l(\alpha)}, \overline{R_l(\alpha)}]$ = [4 α (b4-b3) – b4+2b3 , (-4 α(b8-b7)) –b7+2b8]
(4).$[\overline{S_l(\alpha)}, \overline{S_l(\alpha)}]$ = [-4 α (b5-b4) +b5 , (-4 α(b7-b6)) +b7]

3.Ranking of Intuitionistic Decagonal Fuzzy number

3.1. ExistingRanking technique for Intuitionistic Decagonal fuzzy number:

If A = $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}; b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10})$is a decagonal fuzzy number then we define

$$A = \frac{(-a_1+2a_2+2a_3+2a_4-a_5-a_6+2a_7+2a_8+2a_9-a_{10})+(2b_1+3\,b_4-b_5+2b_6+3b_9-b_{10})}{4},$$

an accuracy function of A, to defuzzify the given number

$$A = \frac{1}{4}\left[\int_0^1 (4\alpha(a_2- a_1) + a_1 - 4\alpha(a_{10}- a_9) + a_{10})\, d\alpha \right.$$
$$+ \int_0^1 (4\alpha(a_3 - a_2) + 2a_2 - a_3 - 4\alpha(a_9 - a_8) + 2a_9 - a_8) d\alpha$$
$$+ \int_0^1 (4\alpha(a_4 - a_3) + 3a_3 - 2a_4 - 4\alpha(a_8 - a_7) + 3a_8 - 2a_7) d\alpha$$
$$\left. + \int_0^1 (4\alpha(a_5 - a_4) + 4a_4 - 3a_5 - 4\alpha(a_7 - a_6) + 4a_7 - 3a_6) d\alpha \right]$$

$$= \frac{1}{4}\left[\int_0^1 (4\alpha(b_2- b_1) + 4\,b_1 - 3b_2 + 4\alpha(b_{10}- b_9) - 3b_{10} + 4b_9)\, d\alpha \right.$$
$$+ \int_0^1 (4\alpha(b_3 - b_2) + 3b_2 - 2b_3 - 4\alpha(b_9 - b_8) + 3b_9 - 2b_8) d\alpha$$
$$+ \int_0^1 (4\alpha(b_4 - b_3) + 2b_3 - b_4 - 4\alpha(b_8 - b_7) + 2b_8 - b_7) d\alpha$$
$$\left. + \int_0^1 (-4\alpha(b_5 - b_4) + b_5 - 4\alpha(b_7 - b_6) + b_7) d\alpha \right]$$

$$A = \frac{1}{4}(-a_1 + 2a_2 + 2a_3 + 2a_4 - a_5 - a_6 + 2a_7 + 2a_8 + 2a_9 - a_{10}) + \frac{1}{4}(2b_1 + 3\, b_4 - b_5 + 2b_6 + 3b_9 - b_{10})$$

$$A = \frac{(-a_1 + 2a_2 + 2a_3 + 2a_4 - a_5 - a_6 + 2a_7 + 2a_8 + 2a_9 - a_{10}) + (2b_1 + 3\, b_4 - b_5 + 2b_6 + 3b_9 - b_{10})}{4}$$

3.2. New Ranking technique for Intuitionistic Decagonal fuzzy number:

For membership function

$$R(\tilde{a}) = \frac{1}{2}\left[\int_0^{0.25} (4\alpha(a_2 - a_1) + a_1, -4\alpha(a_{10} - a_9) + a_{10})\,d\alpha + \int_{0.25}^{0.5} (4\alpha(a_3 - a_2) + 2a_2 - a_3, -4\alpha(a_9 - a_8) + 2a_9 - a_8)\,d\alpha + \int_{0.5}^{0.75} (4\alpha(a_4 - a_3) + 3a_3 - 2a_4, -4\alpha(a_8 - a_7) + 3a_8 - 2a_7)\,d\alpha + \int_{0.75}^{1} (4\alpha(a_5 - a_4) + 4a_4 - 3a_5, -4\alpha(a_7 - a_6) + 4a_7 - 3a_6)\,d\alpha\right]$$

$$R(\tilde{a}) = \frac{1}{2}(0.125a_1 + 0.25a_2 + 0.25a_3 + 0.25a_4 + 0.125a_5 + 0.125a_6 + 0.25a_7 + 0.25a_8 + 0.25a_9 + 0.125a10$$

For non-membership function

$$R(\tilde{a}1) = \frac{1}{2}\left[\int_0^{0.25} (4\alpha(b_2 - b_1) + 4b_1 - 3b_2, 4\alpha(b_{10} - b_9) - 3b_{10} + 4b_9)\,d\alpha + \int_{0.25}^{0.5} (4\alpha(b_3 - b_2) + 3b_2 - 2b_3, -4\alpha(b_9 - b_8) + 3b_9 - 2b_8)\,d\alpha + \int_{0.5}^{0.75} (4\alpha(b_4 - b_3) + 2b_3 - b_4, -4\alpha(b_8 - b_7) + 2b_8 - b_7)\,d\alpha + \int_{0.75}^{1} (-4\alpha(b_5 - b_4) + b_5, -4\alpha(b_7 - b_6) + b_7)\,d\alpha\right]$$

$$R(\tilde{a}1) = \frac{1}{2}[0.875b_1 - 0.25b_2 - 0.25b_3 + 1.25b_4 - 0.625b_5 + 0.875b_6 - 0.25b_7 - 0.25b_8 + 1.25a_9 - 0.625a_{10}]$$

3.2 RANKING OF INTUITIONISTICDECAGONAL FUZZY NUMBERS

The ranking function of DecagonalIntuitionistic Fuzzy Number

$$\check{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10})(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10})$$

maps the set of all Fuzzy numbers to a set of real numbers defined as

$R(\check{A})$=Max[$R(\tilde{a})$, $R(\tilde{a}1)$]

4.NUMERICAL EXAMPLE:

4.1 Crisp Model

ORIGIN	DESTINATION			SUPPLY
1	11	78	6	245
2	48	83	48	305
3	60	94	28	399
DEMAND	340	372	237	

By VAM, the optimal solution is

11 245	78	66
48 95	83 210	48
60	94 162	28 237

TRANSPORTATION COST = 28*237+94*162+83*210+48*94+11*245=**46,549**
Total number of allocated cells :5
m+n-1=5
By Modimethod, every $d_{ij}>0$, then current basic feasible solution is **optimal**

4.2 Using Existing Ranking Procedure

Consider Supplies,Transportation costs and Demands are Decagonal Intuitionistic Fuzzy Number.

ORIGIN	DESTINATION			SUPPLY
1	(1,4,7,8,9, 11,12,15,18,21: 0,2,3,4,7, 11,12,15,18,21)	(70,72,73,74,78, 80,81,82,83,85; 68,70,71,72,73, 80,81,82,83,85)	(58,59,62,63,65, 66,68,72,73,75; 57,58,59,60,62, 66,68,72,73,75)	(205,220,225,230,245, 255,260,265 270,275; 205,220,225,230,240, 255,265,270,275,290)
2	(33,35,36,42,45, 51,57,59,61,62; 30.31,32,34,35, 51,57,59,61,62)	(75,76,77,80,83, 81,87,89,91,92; 74,75,77,78,80, 84,87,89,91,93)	(40,41,44,47,48, 49,50,51,52,53; 39,40,42,44,46, 49,50,51,52,53)	(286,288,295,298,303, 308,310,315,320,325; 285,288,295,298,303, 309,310,315,320,326)
3	(55,56,57,58,59, 60,61,62,63,64; 54,55,56,57,58, 60,63,64,65,66)	(80,83,84,88,91, 95,98,101,107,110; 79,81,83,85,87, 95,98,102,108,112)	(20,21,24,25,27, 29,31,34,35,37; 20,22,24,25,27, 29,30,36,37,38)	(381,384,390,397,398, 401, 405,410,412,415; 378,384,390,397,398, 401, 405,410,412,416)
DEMAND	(320,323,327,330,345, 348,349,351,352,355; 320,323,327,330,343, 348,349,351,352,355)	(359,360,365,366,372, 375,378,379,380,387; 358,361,365,366,371, 375,378,379,380,388)	(184,194,197,201,215, 230,240,300,302,307; 184,193,196,208,210, 230,253,300,310,312)	

$$A = \frac{(-a_1+2a_2+2a_3+2a_4-a_5-a_6+2a_7+2a_8+2a_9-a_{10})+(2b_1+3\,b_4-b_5+2b_6+3b_9-b_{10})}{4}$$

ORIGIN	DESTINATION			SUPPLY
1	36.5	304.75	259.5	966.25
2	184.75	329.75	186.25	1210.75
3	236	368.5	111.5	1593
DEMAND	1345	1477	948	

By Vogel's approximation

ORIGIN	DESTINATION		
1	36.5 966.25	304.75	259.5
2	184.75 378.75	329.75 832	186.25
3	236	368.5 645	111.5 948

TRANSPORTATION COST:
46.5*966.25+329.75*832+184.75*378.75+368.5*645+111.5*948 =722,978.6875
Total number of allocated cells :5
m+n-1=5
By Modimethod , every $d_{ij}>0$, then current basic feasible solution is **optimal**

4.3 Using New Ranking Technique

Now the new ranking technique is used for the same problem given in section 4.2

R(ã)
$=\frac{1}{2}(0.125a_1 + 0.25a_2 + 0.25a_3 + 0.25a_4 + 0.125a_5 + 0.125a_6 + 0.25a_7 + 0.25a_8 + 0.25a_9 +$
0.125a10

$a_1 = 1\ a_2 = 4\ a_3 = 7\ a_4 = 8\ a_5 = 9\ a_6 = 11\ a_7 = 12\ a_8 = 15\ a_9 = 18\ a_{10} = 21$

$R(\tilde{a}_{11})=$
$\frac{1}{2}(0.125 * 1 + 0.25 * 4 + 0.25 * 7 + 0.25 * 8 + 0.125 * 9 + 0.125 * 11 + 0.25 * 12 + 0.25 *$
15+0.25 18+0.125* 21*
$R(\tilde{a}_{11})= 10.6$

$$R(\tilde{a}1) = \frac{1}{2}[0.875b_1 - 0.25b_2 - 0.25b_3 + 1.25b_4 - 0.625b_5 + 0.875b_6 - 0.25b_7 - 0.25b_8 + 1.25a_9 - 0.625a_{10}]$$

$$[b_1 = 0\ b_2 = 2\ b_3 = 3\ b_4 = 4\ b_5 = 7 b_6 = 11 b_7 = 12 b_8 = 15 a_9 = 18 a_{10} = 21]$$

$R(\tilde{a}1_{11})=\frac{1}{2}[0.875 * 0 - 0.25 * 2 - 0.25 * 3 + 1.25 * 4 - 0.625 * 7 + 0.875 * 11 - 0.25 *$
12– 0.25 15+1.25* 18–0.625* 21*

$R(\tilde{a}1_{11})$ =-4.3125

ORIGIN	DESTINATION			SUPPLY
1	10.5 -4.3125	77.5 27.5625	66 20.9375	245 74.0625
2	47.5 7.8125	83 28.5	47 15.4375	304.5 118.5625
3	59 21.0625	93.5 28.3125	28 5.0625	399 280.34375
DEMAND	339.5 236.5625	371.5 261.0625	237.5 49.625	

$R(\widetilde{A_{11}})$=Max[$R(\tilde{a}_{11}), R(\tilde{a}1_{11})$]
$R(\widetilde{A_{11}})$=Max[10.5,-4.3125] = 10.62

ORIGIN	DESTINATION			SUPPLY
1	10.5	77.5	66	245
2	47.5	83	47	304.5
3	59	93.5	28	399
DEMAND	339.5	371.5	237.5	

By VAM, the optimal solution is

ORIGIN	DESTINATION		
1	10.5 [245]	77.5	66
2	47.5 [94.5]	83 [210]	47
3	59	93.5 [161.5]	28 [237.5]

TRANSPORTATION COST: 28*237.5+93.5*161.5+83*210+47.5*94.5+10.5*245=**46,241.5**

Total number of allocated cells :5 ,m+n-1=5

By Modimethod,every d_{ij}>0, then current basic feasible solution is **optimal**

5. Conclusion:

In this research paper a new ranking procedure of intuitionistic decagonal fuzzy number is defined and a problem is solved using the ranking technique. This paper concludes that this new ranking technique will giveminimum transportation cost when compared to the already existing formula.

References

1. AbhinavBansal (2011) Trapezoidal Fuzzy numbers (a,b,c,d):Arithmetic behavior. International Journal of Physical and Mathematical Sciences, ISSN-2010-1791.
2. C.H. Cheng, A new approach for ranking fuzzy numbers by distance method, Fuzzy Sets and Systems, 95 (1998), 307–317
3. S.Elizabeth and L.Sujatha, Project scheduling method using Triangular Intuitionistic fuzzy numbers and Triangular fuzzy numbers, 9(4) (2015), 185 - 198
4. J.JonArockiaraj and N.Sivasankari,A Decagonal Fuzzy Number and its Vertex Method,International Journal of Mathematics And its Applications,4(2016),283-292.
5. P. Malini and M.Ananthanarayanan, solving fuzzy transportation problem using ranking of octagonal fuzzy numbers, International Journal of Pure and Applied Mathematics,110(2016), 275-282.
6. G.Menaka, Ranking Of Octagonal Intuitionistic Fuzzy Numbers, IOSR Journal of Mathematics,13(2017),63-71.
7. MichealO'hEigeartaigh, A fuzzy transportation algorithm, Fuzzy Sets and System, 8 (1982), 235–243.
8. NagoorGani.A, Abbas. S,(2013) "A New method for solving in Fuzzy Transportation Problem", Applied Mathematics Sciences, vol.7,No.28, 1357 – 1365.
9. O'heigeartaigh.H,(1982) "A Fuzzy Transportation Algorithm" Fuzzy Sets and Systems, 235-243.
10. P. Pandian and G. Natarajan, A new algorithm for finding a fuzzy optimal solution for fuzzy transportation, Applied Mathematical Sciences, 4 (2010), 79–90.
11. P. PhaniBushanRao and N. Ravishankar, Ranking fuzzy numbers with a distance method using circumcenter of centroids and an Index of Modality, Advances in Fuzzy System, 10 (2011), 1155–1161.
12. A.praveenprakash ,N.JoseParvinPraveena and A. Rajkumar ,A New Intuitionistic Decagonal fuzzy number and its application,International Journal of Computer Application,7(2017), 1797-2250.
13. H.J.Zimmermann, Fuzzy Set Theory and its Application, Fourth Edition, Springer, (2011).
14. Zadeh, L.A (1965). "Fuzzy sets."Informationcontrol, 8,338-353.