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On Signed Cycle Domination Number for Some General Graphs

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ABSTRACT: *Nowadays, Domination problems have many applications like resource allocation, network routing, electronic circuits and encoding theory problems. Let $G = (V, E)$ be a graph. A function $f: E \to \{+1, -1\}$ is called a signed cycle dominating function (SCDF) of G if $\sum_{e \in E(C)} f(e) \geq 1$ holds for every induced cycle C of G. The signed cycle domination number of G, denoted by $\gamma'_{sc}(G)$, is defined as $\gamma'_{sc}(G) = min\{\sum_{e \in E(G)} f(e) \mid f \text{ is an SCDF of } G\}$. In this paper we obtain some exact values of signed cycle domination number for few classes of graph.*

Keywords: *Signed cycle domination function, Signed cycle domination number, corona product.*

I. INTRODUCTION

The concept of signed cycle domination of graph was introduced by Xu, see [4], which characterized all connected graphs G with $\gamma'_{sc}(G) = |E(G)| - 2$, and determine the exact value of $\gamma'_{sc}(G)$ for tree, fan graph, complete graph, wheel graph, and list two open problems and three conjectures. In [2], Jian Guan et al. proved two conjectures and disproved one conjecture. Later, Xiao Ming Pi [5] characterized all maximal planar graph G with order $n \geq 6$, then $\gamma'_{sc}(G) = n$.

Recently, Sundarakannan and Arumugam proved that the decision problem of SCDN is NP-complete, see [3].

In Signed cycle domination, a finite, undirected and connected simple graph G with vertex set V(G) and edge **set** E(G) are considered. In a graph G the order and size of G denoted by n and m respectively. Let open neighborhood and the closed neighborhood of v is denoted by N(v) and N[v] if $N(v) = \{u \epsilon V: uv \epsilon E\}$ and $N[v] = N(v) \cup \{v\}$. If $S \in V$, then S is an induced subgraph of G and is denoted by $\langle S \rangle$. C is an induced cycle of G if $\langle V(C) \rangle = C$. A **unicyclic** graph is a connected graph containing exactly one cycle.

A **m-partite graph** is a graph whose vertices can be partitioned into m different independent sets $\{V_1, V_2, \ldots, V_m\}$. A complete m-partite graph is a m-partite graph in which there is an edge between every pair of vertices from different independent sets V_i and V_j, $i \neq j$; it is denoted by $K_{\{r_1, r_2, \ldots, r_m\}}$, where $r_i = |V_i|$.

The graph $G \circ H$ is obtained by taking |V(G)| copies of H and joining the i-th vertex of G to all vertices in the i-th copy of H. $G \circ H$ is called the **corona** of G and H. It is not commutative, i.e., $G \circ H \neq H \circ G$.

[1] Let $G = (V, E)$ be a graph; a function $f: E \to \{+1, -1\}$ is called a **signed cycle dominating function (SCDF)** of G if $\sum_{e \in E(C)} f(e) \geq 1$ holds for every induced cycle C of G. The **signed cycle domination number (SCDN)** of G, denoted by $\gamma'_{sc}(G)$, is defined as $\gamma'_{sc}(G) = \min\{\sum_{e \in E(G)} f(e) \mid f \text{ is an SCDF of } G\}$.

II. Main Results

In this section we present signed cycle domination number of some classes of graphs such as cycle, $K_n \circ \overline{K_s}$, K-vertex disjoint cycles, unicyclic graph.

Theorem 1. Let G be a cycle with $n \geq 3$, then signed cycle domination number

$$\gamma'_{sc}(C_n) = \begin{cases} 1 & if\ n\ is\ odd \\ 2 & if\ n\ is\ even \end{cases}$$

Proof: Let G be a cycle of length $n \geq 3$. Let the edges of G, in order, be $e_1, e_2, e_3, \ldots e_n$. The result is obvious when n=3 or n=4. Suppose that $n \geq 5$.

We consider two cases:

Case 1 Let G be the cycle with odd length, then signed cycle domination function (SCDF) f of C_n is defined as follows:

$$f(e) = \begin{cases} -1 & if\ e \in \left\{e_1, e_2, e_3, \dots, e_{\frac{n-1}{2}}\right\} \\ +1 & otherwise \end{cases}$$

The signed cycle domination number is
$\gamma_{sc}^{'}(C_n) = min\{\sum_{e\in E(C_n)} f(e) \mid f\ is\ an\ SCDF\ of\ G\}$
$\gamma_{sc}^{'}(C_n) = min\{\sum_{e\in E_1(C_n)} f(e) - \sum_{e\in E_2(C_n)} f(e) \mid f\ is\ an\ SCDF\ of\ G\}$

Where E_1 denotes the set of edges of C_n with $f(e) = 1$, and E_2 denotes the set of edges of C_n with $f(e) = -1$. As
$\sum_{e\in E(C)} f(e) \geq 1$, $|E_2| = \frac{n-1}{2}$,
$|E_1| = |E(C_n)| - |E_2| = n - \frac{n-1}{2} = \frac{n+1}{2}$.
$\gamma_{sc}^{'}(C_n) \leq \{\sum_{e\in E_1(C_n)} f(e) - \sum_{e\in E_2(C_n)} f(e)\}$,

$\gamma_{sc}^{'}(C_n) \leq \frac{n+1}{2} - \frac{n-1}{2} \leq 1.$

By the definition of any SCDF
$\sum_{e\in E(C_n)} f(e) \geq 1$

Hence $\gamma_{sc}^{'}(C_n) = 1$, when n is odd.

Case 2. Let G be the cycle with even length, then signed cycle domination function (SCDF) f of C_n is defined as follows:

$$f(e) = \begin{cases} -1 & if\ e \in \{e_1, e_2, e_3, \dots, e_{\frac{n}{2}-1}\} \\ +1 & otherwise \end{cases}$$

As in the previous case, let E_1 denote the set of edges of C_n with $f(e) = 1$, and E_2 denotes the set of edges of C_n with $f(e) = -1$.

Clearly, $|E_2| = (n/2) - 1$,
$|E_1| = |E(C_n)| - |E_2| \quad = n - \left((n/2) - 1\right) = (n/2) + 1.$
Then $\gamma_{sc}^{'}(C_n) \leq (n/2) + 1 - ((n/2) - 1) \leq 2.$
Next we prove that $\gamma_{sc}^{'}(C_n) \geq 2.$
Suppose $\gamma_{sc}^{'}(C_n) < 2$, $\left\{\sum_{e\in E_1} f'(e) - \sum_{e\in E_2} f'(e)\right\} = 1$, that is, $n_1 + n_2 = n$. Where, $n_i = |E_i|$ and $n_1 - n_2 = 1$. This implies $2n_1 = n + 1$, it contradicts as right hand side is odd and left hand side is even. Hence $\gamma_{sc}^{'}(C_n) = 2$, where n is even.

Theorem 2. The signed cycle domination number for complete graph [1] is $\binom{n}{2} - 2\lfloor n/2 \rfloor$.

Theorem 3. Let r and s be positive integers where $r \geq 3$ and $s \geq 1$, then
$\gamma_{sc}^{'}(K_r \circ \overline{K_s}) = \binom{r}{2} - 2(\lfloor r/2 \rfloor) - rs$, Where $\circ$ denotes the corona product.
Proof: Let K_r be the complete graph with r vertices and $\overline{K_s}$ be the complement of K_s. The graph $K_r \circ \overline{K_s}$, obtained by taking r copies of $\overline{K_s}$, say $V_1, V_2, \dots, V_r$ and joining the i-th vertex of K_r to all vertices in the i-th copy of $\overline{K_s}$, that is, to V_i.
We suppose that $r \geq 3$ and $s \geq 1$.
Now assign the value -1 to $\lfloor r/2 \rfloor$ independent edges of K_r and all the edges incident with the end vertices of $K_r \circ \overline{K_s}$. As this assignment is an optimal one,
$\gamma_{sc}^{'}(K_r \circ \overline{K_s}) = \binom{r}{2} - 2(\lfloor r/2 \rfloor) - rs.$

Theorem 4: Let $G \circ H$ be any graph. Then Signed cycle domination number for $G \circ H$ is
$\gamma_{sc}^{'}(G \circ H\ ) \leq \gamma_{sc}^{'}(G) + n\,\gamma_{sc}^{'}(H) + mn$
Proof: If number of vertices in G is $|V(G)| = n$, number of vertices in H is $|V(H)| = m$

Let Q_i be the number of edges which associates each vertex v_i in G to H_i.
Q=$\sum_{i=1}^{n} Q_i = nm$ and total number of vertices in $|G \circ H| = n + mn$.
$\gamma_{sc}^{'}(G) = \min\{\sum_{e \in E(G)} f(e) \mid f \text{ is an SCDF of } G\}$
$\gamma_{sc}^{'}(H) = \min\{\sum_{e \in E(H)} f(e) \mid f \text{ is an SCDF of } H\}$
By the definition of corona, ith copy of G is adjacent with all vertices in ith copy of H. Therefore, there exist n copies of H. i.e., $n\,\gamma_{sc}^{'}(H)$ exist.
Signed cycle domination number for $G \circ H$ is less than or equal to sum of the SCDN of graph G, SCDN of graph H and Q.
i.e., $\gamma_{sc}^{'}(G \circ H) \leq \gamma_{sc}^{'}(G) + n\,\gamma_{sc}^{'}(H) + mn$

Theorem 5. Let G be a connected graph in which each of its block is either a cycle or an edge and further, the cycles are vertex disjoint. If the disjoint cycles are $C_{l_1}, C_{l_2}, C_{l_3}, \ldots, C_{l_k}$, $\sum_{i=1}^{k} l_i = r$, then $\gamma_{sc}^{'}(G) = \sum_{i=1}^{k} \gamma_{sc}^{'}(C_i) - (n + k - r - 1)$.
Proof. Let f be a minimum SCDF. As each edge of $E(G) \setminus \bigcup_{i=1}^{k} E(C_i)$ is not on a cycle, let $f(e) = -1$. Let $f_i = \{f | C_i$ be a minimum SCDF on $C_i\}$, then $\gamma_{sc}^{'}(G) = \sum_{i=1}^{k} \gamma_{sc}^{'}(C_i) - (n + k - r - 1)$.

Corollary 6. Let G be a unicyclic graph and let C be the cycle of length l in G. Then

$$\gamma_{sc}^{'}(G) = \begin{cases} l - n + 2 & if\ l\ is\ even \\ l - n + 1 & if\ l\ is\ odd \end{cases}$$

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RELATIVE CONTROLLABILITY OF FRACTIONAL STOCHASTIC DELAY DYNAMICAL SYSTEMS WITH DISTRIBUTED DELAYS IN CONTROL

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ABSTRACT: *In this paper, we study the relative controllability of fractional stochastic delay dynamical systems with distributed delays in control, which involves Caputo derivatives. A necessary and sufficient condition for relative controllability of linear stochastic delay dynamical system with distributed delays in control has derived based on controllability Grammian matrix is defined by Mittag-Leffler matrix function. Sufficient condition for relative controllability of thecorresponding nonlinear stochastic delay dynamical system has established by using Schauder's fixed point theorem. Finally, a numerical example has given to verify the results.*

Keywords: *Controllability, Stochastic delay systems, Mittag-Leffler matrix function,Fractional differential equations.*

1. Introduction

Fractional delay dynamical systems are an important kind of fractional order systems in real life. In recent years, some authors pay attention to the study about the fractional delay dynamical systems [1-7]. Fractional models are more accurate than integer models. Fractional calculus provides an excellent instrument for the description of systems with memory and hereditary properties. At the same time, the fluctuations in nature can be captured only by adding random elements into the differential equations, which are called stochastic differential equations [8-14]. In particular, time delay frequently exists in control [15-18].

For the systems we study in this paper, we consider two factors (controllability of fractional delay systems, and stochastic) synchronously. The results of this paper should be useful.

2. Preliminaries

Let $\alpha, \beta > 0$, with $n-1 < \alpha < n$, $n-1 < \beta < n$and $n \in N$, $D$is the usual differential operator. Let R^m be the m-dimensional Euclidean space, $R_+ = [0, \infty)$, and suppose $f \in L^1(R_+)$. The following definitions and properties are well known, for $\alpha, \beta > 0$ and $f$as a suitable function [8, 9].

(a) Riemann-Liouville fractional operators:

$$(I_{a+}^{n-\alpha} f)(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{(\alpha-1)} f(t)dt,$$
$$(D_{0+}^{\alpha} f)(x) = D^n (I_{a+}^{n-\alpha} f)(x)$$

(b) Caputo fractional derivative:

$$\left({}^{C}D_{0+}^{\alpha} f\right)(x) = (I_{a+}^{n-\alpha} D^n f)(x),$$

In particular $I_{0+}^{\alpha}\, {}^{c}D_{0+}^{\alpha} f(t) = f(t) - f(0), (0 < \alpha < 1)$.

The following is a well-known relation, for finite interval $[a, b] \in R_+$

$$(D_{0+}^{\alpha} f)(x) = \left({}^{C}D_{0+}^{\alpha} f\right)(x) + \sum_{k=0}^{n-1} \frac{f^{(k)}(a)}{\Gamma(1+k+\alpha)} (x-a)^{k-\alpha}, n = R(\alpha) + 1$$

The Laplace transform of the Caputo fractional derivative is

$$\mathcal{L}\{\,{}^{C}D^{\alpha}_{0+}f(t)\} = s^{\alpha}F(s) - \sum_{k=0}^{n-1} f^{(k)}(0^{+})s^{\alpha-1-k}.$$

From the above we notice that, the Riemann-Liouville fractional derivatives have singularity at zero and the fractional differential equations in the Riemann-Liouville sense require initial conditions of special form lacking physical interpretation. To overcome this difficulty Caputo introduced a new definition of fractional derivative but in general, both the Riemann-Liouville and the Caputo fractional operators possess neither semigroup nor commutative properties, which are inherent to the derivatives on integer order. Due to this fact, the concept of sequential fractional differential equations are discussed in Kilbas et al. [8] and Miller and Ross [9].

(c) Linear Sequential Derivative:

For $n \in N$ the sequential fractional derivative for suitable function f is defined by

$$f^{(k\alpha)} := (D^{k\alpha}f)(x) = (D^{\alpha}D^{(k-1)\alpha}f)(x),$$

where $k = 1, \ldots, n$, $(D^{\alpha}f)(x) = f(x)$, and D^{α} is any fractional differential operator, here we mention it as ${}^{C}D^{\alpha}_{0+}$.

(d) Mittag-Leffler Function

$$E_{\alpha,\beta}(y) = \sum_{k=0}^{\infty} \frac{y^k}{\Gamma(ka+\beta)}, \qquad \alpha, \beta > 0.$$

The general Mittag-Leffler function satisfies

$$\int_0^{\infty} e^{-t}t^{\beta-1}E_{\alpha,\beta}(t^{\alpha}y)dt = \frac{1}{1-y}, \quad |y| < 1.$$

The Laplace transform of $E_{\alpha,\beta}(y)$ follows from the integral

$$\int_0^{\infty} e^{-st}t^{\beta-1}E_{\alpha,\beta}(\pm at^{\alpha}y)dt = \frac{s^{\alpha-\beta}}{s \mp a}.$$

That is

$$\mathcal{L}\{t^{\beta-1}E_{\alpha,\beta}(\pm at^{\alpha})\} = \frac{s^{\alpha-\beta}}{s \mp a},$$

For $\mathcal{R}(s) > |a|^{\frac{1}{\alpha}}$ and $R(\beta) > 0$. In particular for $\beta = 1$,

$$E_{\alpha,1}(\lambda y^{\alpha}) = E_{\alpha}(\lambda y^{\alpha}) = \sum_{k=0}^{\infty} \frac{\lambda^k y^{k\alpha}}{\Gamma(\alpha k+1)}, \quad \lambda, y \in C$$

Have the interesting property ${}^{C}D^{\alpha}_{0+}E_{\alpha}(\lambda y^{\alpha}) = \lambda E(\lambda y^{\alpha})$and

$$\mathcal{L}\{E_{\alpha}(\pm at^{\alpha})\} = \frac{s^{\alpha-\beta}}{s \mp a}, \qquad for\ \beta = 1.$$

For brevity of notation let us take I^{α}_{0+} as $I^{\alpha}$and ${}^{C}D^{\alpha}_{0+}$ as ${}^{C}D^{\alpha}$ and the fractional derivative is taken as Caputo sense.

3. Linear system

Consider the linear fractional stochastic delay dynamical system with distributed delays in controlof the form

$$cD^{\alpha}x(t) = Ax(t) + Bx(t-h) + \int_{-h}^{0} d_{\tau}C(t,\tau)u(t+\tau) + \sigma(t)\frac{d\omega(t)}{dt}, t \in J := [0,T]$$

$$x(t) = \varphi(t), x'(t) = \varphi'(t), \qquad (3.1)$$
$$u(t) = \psi(t), \qquad -h < t \le 0.$$

Where$1 < \alpha \le 2$,$x \in R^n$, $u \in R^m$, A and B are $n \times n$matrices, $C(t,\tau)$is an $n \times m$matrix,continuous in $t$for fixed $\tau$and is of bounded variation in τon $[-h,0]$for each $t \in J$.The integral term is in the Lebesgue Stieltjes sense that is denote by the symbol d_{τ}, for function u: $[-h,T] \to R^m$and$t \in J$, we use the symbol u_t to denotes the function on $[-h,0)$ defined by $u_t(\tau) = u(t+\tau)$ for $\tau \in [-h,0)$.

The general solution of system (3.1) can be written as

$$\begin{aligned} x(t) = {} & \Phi_{\alpha}(t)\varphi(0) + t\Phi_{\alpha,2}(t)\varphi'(0) \\ & + B\int_{-h}^{0}(t-s+h)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s+h)\varphi(s)ds \\ & + \int_{0}^{t}(t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)\left[\int_{-h}^{0} d_{\tau}C(t,\tau)u(t+\tau)\right]ds \\ & + \int_{0}^{t}(t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)\left(\int_{0}^{\tau}\sigma(\theta)\,d\omega(\theta)\right)ds. \end{aligned} \qquad (3.2)$$

Now using the well known result of unsymmetric Fubini [19] theorem and change oforder of integration to the last term, we have

$$\begin{aligned} x(t) = {} & \Phi_{\alpha}(t)\varphi(0) + t\Phi_{\alpha,2}(t)\varphi'(0) + B\int_{-h}^{0}(t-s+h)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s+h)\varphi(s)ds \\ & + \int_{-h}^{0} dC_{\tau}\left[\int_{-h}^{0}(t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)C(s,\tau)u(s+\tau)\,ds\right] \\ & + \int_{0}^{t}(t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)\left(\int_{0}^{\tau}\sigma(\theta)\,d\omega(\theta)\right)ds, \end{aligned}$$

$$\begin{aligned} x(t) = {} & \Phi_{\alpha}(t)\varphi(0) + t\Phi_{\alpha,2}(t)\varphi'(0) + B\int_{-h}^{0}(t-s+h)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s+h)\varphi(s)ds \\ & + \int_{-h}^{0} dC_{\tau}\left[\int_{\tau}^{0}\big(t-(s-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(t-(s-\tau)\big)C(s-\tau,\tau)\psi(s)\,ds\right] \\ & + \int_{-h}^{0} dC_{\tau}\left[\int_{0}^{t+\tau}\big(t-(s-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(t-(s-\tau)\big)C(s-\tau,\tau)\right]u(s)ds \\ & + \int_{0}^{t}(t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)\left(\int_{0}^{\tau}\sigma(\theta)\,d\omega(\theta)\right)ds, \end{aligned}$$

$$\begin{aligned} x(t) = {} & x(t;\varphi) + \int_{-h}^{0} dC_{\tau}\left[\int_{\tau}^{0}\big(t-(s-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(t-(s-\tau)\big)C(s-\tau,\tau)\psi(s)\,ds\right] \\ & + \int_{0}^{t} dC_{\tau}\left[\int_{-h}^{0}\big(t-(s-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(t-(s-\tau)\big)d_{\tau}C_{t}(s-\tau,\tau)\right]u(s)ds \\ & + \int_{0}^{t}(t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)\left(\int_{0}^{\tau}\sigma(\theta)\,d\omega(\theta)\right)ds, \end{aligned} \qquad (3.3)$$

where,

$$x(t;\varphi) = \Phi_{\alpha}(t)\varphi(0) + t\Phi_{\alpha,2}(t)\varphi'(0) + B\int_{-h}^{0}(t-s+h)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s+h)\varphi(s)ds.$$

$$C_t(s,\tau) = \begin{cases} C(s,\tau), & s \le t \\ 0, & s > t \end{cases}$$

and dC_{τ} denotes the integration of Lebesgue Stielties sense with respect to the variableτ in the function $C(t,\tau)$.

For our convenience, let us take

$$H(t,s) = \int_{-h}^{0} \left(t-(s-\tau)\right)^{\alpha-1} \Phi_{\alpha,\alpha}\left(t-(s-\tau)\right) d_\tau C_t(s-\tau,\tau). \tag{3.4}$$

Define the controllability Grammian Matrix

$$W(0,T) = \int_0^T H(T,s)H^*(T,s)ds, \tag{3.5}$$

where the $*$ indicates the matrix transpose.

Definition 3.1.

The set $y(t) = \{x(t), u_t\}$is the complete state of the system (3.1) at time t.

Definition 3.2.

System (3.1) is said to be relatively controllable on J if for everycomplete state y(0) and every vector $x_1 \in R^n$ there exists a control $u(t)$ defined on J such that the corresponding solution of the system (3.1) satisfies $x(T) = x_1$.

The following notations that will be used throughout this paper.

Let (Ω, F, P) be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. right continuous and $\mathcal{F}_0$containing all P-null sets).Let $(\Omega, \mathcal{F}, P)$ denotes the complete probability space with a probability measure P on Ω and $\omega(t) = \left(\omega_1(t), \omega_2(t), \cdots, \omega_n(t)\right)^*$ be an n–dimensional Wiener process defined on the probability space. Let $\{\mathcal{F}_t | t \in J\}$ is the filtration generated by $\{\omega(s) : 0 \leq s \leq t\}$ defined on the probability space $(\Omega, \mathcal{F}, P)$. Let $L_2(\Omega, \mathcal{F}_t, R^n \times R^m)$ is the Hilbert space of all $\mathcal{F}_t$– measurable square integrable random variables with values in $R^n \times R^m$. Let $L_2^{\mathcal{F}}(J, R^n)$ is the Hilbert space of all square-integrable and $\mathcal{F}_t$- measurable processes with values inR^n. Let $C_n(J) = C\left(J, L_2(\Omega, \mathcal{F}_t, R^n)\right)$ be the Banach space of continuous maps from J into $L_2(\Omega, \mathcal{F}_t, R^n)$. Define $\boldsymbol{\mathcal{B}} = C_n(J) \times C_m(J)$ be the Banach space of continuous $L_2(\Omega, \mathcal{F}_t, R^n \times R^m)$ valued $\mathcal{F}_t$– adapted square integrable functions $\left(z(t), v(t)\right)$ with norm $\|(z,v)\|^2 = \|z\|^2 + \|v\|^2$, where $\|z\|^2 = \sup_{t\in J} E\|z(t)\|^2$, $E(\cdot)$ denotes the mathematical expectation operator of stochastic process with respect to the given probability measure P and $\mathcal{U}_{ad} : L_2^{\mathcal{F}}(J, R^m)$ is the set of admissible controls.

Theorem 3.1.

The linear control system (3.1)is relatively controllable on $[0, T]$if and only if thecontrollability Grammian matrix $W(0,T) = \int_0^T H(T,s)H^*(T,s)ds$is positive definite, forsome $T > 0$.

Proof:

We first show that the necessity. Suppose that$W$ is not positive definite. Then$W$is singular and so its inverse is does not exist, and there exists a nonzero y such that

$$y^* W y = y^* \int_0^T H(T,s)H^*(T,s)y ds = 0,$$

and hence, for$s \in [0, T]$

$$y^* H(T,s) = y^* \int_{-h}^{0} \left(T-(s-\tau)\right)^{\alpha-1} \Phi_{\alpha,\alpha}\left(T-(s-\tau)\right) d_\tau C_T(s-\tau,\tau) = 0. \tag{3.6}$$

Now choose $y = x(T\,;\,\varphi)$and take $\psi(s) = 0$. Since system (3.1) is relatively controllable, there exists a control $u \in C(J)$ such that if steers the complete state $y(0) = \{\varphi(0), \varphi'(0), \psi(s)\}$to the origin in the interval J, it follow that

$$\begin{aligned} x(T) = x(T;\varphi) &+ \int_{-h}^{0} dC_\tau \left[\int_\tau^0 \left(T-(s-\tau)\right)^{\alpha-1} \Phi_{\alpha,\alpha}\left(T-(s-\tau)\right) C(s-\tau,\tau)\psi(s)\, ds\right] \\ &+ \int_0^T \left[\int_{-h}^{0} \left(T-(s-\tau)\right)^{\alpha-1} \Phi_{\alpha,\alpha}\left(T-(s-\tau)\right) d_\tau C_T(s-\tau,\tau)\right] u(s) ds \\ &+ \int_0^T (T-s)^{\alpha-1} \Phi_{\alpha,\alpha}\,(T-s) \left(\int_0^\tau \sigma(\theta)\, d\omega(\theta)\right) ds, \end{aligned}$$

$$x(T) = y + \int_0^T \left[\int_{-h}^0 \left(T-(s-\tau)\right)^{\alpha-1}\Phi_{\alpha,\alpha}\left(T-(s-\tau)\right)d_\tau C_T(s-\tau,\tau)\right]u(s)ds + \int_0^T (T-s)^{\alpha-1}\Phi_{\alpha,\alpha}(T-s)\left(\int_0^\tau \sigma(\theta)\,d\omega(\theta)\right)ds$$
$$= 0.$$

Thus,

$$y^*y + \int_0^T y^*H(T,s)u(s)ds = 0. \tag{3.7}$$

It follows from (3.6) and (3.7) that $y^*y = 0$. This is a contradiction to $y \neq 0$. Thus W is nonsingular.

Next, we show that the sufficiency. Suppose that W is positive definite, that is, it is nonsingular and so its inverse is well-defined. Define the control function as

$$u(t) = H^*(T,s)W^{-1}\Bigg[x_1 - x(T,\varphi) - \int_{-h}^0 dC_\tau\left[\int_\tau^0 \left(T-(s-\tau)\right)^{\alpha-1}\Phi_{\alpha,\alpha}\left(T-(s-\tau)\right)C(s-\tau,\tau)\psi(s)ds\right] - \int_0^t (t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)\int_0^\tau \sigma(\theta)d\omega(\theta)ds\Bigg], \tag{3.8}$$

where the complete state $y(0)$ and the vector $x_1 \in R^n$ are choosenarbitrarily. Substituting $u(t)$ in (3.3) and using (3.4) we have

$$x(T) = x(T;\varphi) + \int_{-h}^0 dC_\tau\left[\int_\tau^0 \left(T-(s-\tau)\right)^{\alpha-1}\Phi_{\alpha,\alpha}\left(T-(s-\tau)\right)C(s-\tau,\tau)\psi(s)\,ds\right] + \int_0^T\left[\int_{-h}^0 \left(T-(s-\tau)\right)^{\alpha-1}\Phi_{\alpha,\alpha}\left(T-(s-\tau)\right)d_\tau C_T(s-\tau,\tau)\right] \times H^*(T,s)W^{-1}\Bigg[x_1 - x(T,\varphi) - \int_{-h}^0 dC_\tau\left[\int_\tau^0 \left(T-(s-\tau)\right)^{\alpha-1}\Phi_{\alpha,\alpha}\left(T-(s-\tau)\right)C(s-\tau,\tau)\psi(s)ds\right] - \int_0^t (t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)\int_0^\tau \sigma(\theta)d\omega(\theta)ds\Bigg]ds + \int_0^T (T-s)^{\alpha-1}\Phi_{\alpha,\alpha}(T-s)\left(\int_0^\tau \sigma(\theta)\,d\omega(\theta)\right)ds,$$

$$x(T) = x(T;\varphi) + \int_{-h}^0 dC_\tau\left[\int_\tau^0 \left(T-(s-\tau)\right)^{\alpha-1}\Phi_{\alpha,\alpha}\left(T-(s-\tau)\right)C(s-\tau,\tau)\psi(s)\,ds\right] + \int_0^T\Bigg[\int_{-h}^0 \left(T-(s-\tau)\right)^{\alpha-1}\Phi_{\alpha,\alpha}\left(T-(s-\tau)\right)d_\tau C_T(s-\tau,\tau) \times H^*(T,s)W^{-1}[x_1 - x(T,\varphi) - \int_{-h}^0 dC_\tau\left[\int_\tau^0 \left(T-(s-\tau)\right)^{\alpha-1}\Phi_{\alpha,\alpha}\left(T-(s-\tau)\right)C(s-\tau,\tau)\psi(s)\,ds\right] - \int_0^T (T-s)^{\alpha-1}\Phi_{\alpha,\alpha}(T-s)\int_0^\tau \sigma(\theta)d\omega(\theta)ds.\Bigg]ds\Bigg] + \int_0^T (T-s)^{\alpha-1}\Phi_{\alpha,\alpha}(T-s)\left(\int_0^\tau \sigma(\theta)\,d\omega(\theta)\right)ds,$$

$x(T) = x_1$.

This means that control $u(t)$ transfer the initial state $y(0)$ to the desired vector $x_1 \in R^n$ at time T. Hence the system (3.1) is relatively controllable.

For convenience, let us introduce the following constants

$m_1 = \sup_{t\in J}\left\|\Phi_{\alpha,\alpha}\big(t-(s-\tau)\big)\right\|^2, m_2 = \sup_{t\in J}\left\|\Phi_{\alpha,\alpha}(t-s)\right\|^2, m_3 = \max\{6\|H^*(T,s)\|^2T^2, 1\},$

$$m_4 = 4\times 6\|H^*(T,s)\|^2\|W^{-1}\|^2\left[\|x_1\|^2 + \|x(T,\varphi)\|^2 + \left\|\int_{-h}^{0} dC_\tau\left[\int_\tau^0 \big(T-(s-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(T-(s-\tau)\big)C(s-\tau,\tau)\psi(s)\,ds\right]\right\|^2\right]$$

$$m_5 = 4\times 6\left[\|x_1\|^2 + \|x(T,\varphi)\|^2 + \left\|\int_{-h}^{0} dC_\tau\left[\int_\tau^0 \big(T-(s-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(T-(s-\tau)\big)C(s-\tau,\tau)\psi(s)\,ds\right]\right\|^2\right]$$

$$m_6 = 4\times 6\|H^*(T,s)\|^2\|W^{-1}\|^2\left(T^2m_2L + \frac{T^{2\alpha+1}}{\alpha^2}m_1M_\sigma M\right)\left(1+\sup_{t\in J}E\|z(t)\|^2 + \sup_{t\in J}E\|v(t)\|^2\right)$$

$$m_7 = 4\times 6\left(T^2m_2L + \frac{T^{2\alpha+1}}{\alpha^2}m_1M_\sigma M\right)\left(1+\sup_{t\in J}E\|z(t)\|^2 + \sup_{t\in J}E\|v(t)\|^2\right)$$

$$N(T) = \int_{-h}^{0} dC_\tau\left[\int_\tau^0 \big(T-(s-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(T-(s-\tau)\big)C(s-\tau,\tau)\psi(s)\,ds\right]$$

4. Nonlinear system

Consider the nonlinear fractional stochastic delay dynamical system with distributed delays in control of the form

$$\begin{aligned} cD^\alpha x(t) &= Ax(t) + Bx(t-h) + \int_{-h}^{0} d_\tau C(t,\tau)u(t+\tau) + \sigma(t,x(t),x(t-h))\frac{d\omega(t)}{dt} + f\big(t,x(t),x(t-h)\big), t\in J := [0,T] \\ x(t) &= \varphi(t), x'(t) = \varphi'(t), \\ u(t) &= \psi(t), \qquad -h < t \le 0 \end{aligned} \tag{4.1}$$

where $1<\alpha\le 2, x\in R^n,\ u\in R^m$, A and B are $n\times n$matrices, $C(t,\tau)$is an $n\times m$matrix, continuous in t for fixed $\tau$and is of bounded variation in τon $[-h,0]$ for each $t\in J$. The integral term is in the Lebesgue Stieltjes sense that is denoted by the symbol d_τ, for function $u:[-h,T]\to R^m$and $t\in J$, we use the symbol u_t to denotes the function on $[-h,0)$ defined by $u_t(\tau)=u(t+\tau)$ for $\tau\in[-h,0)$.The nonlinear function $f: J\times R^n\times R^n\to R^n$ is continuous.

Then the solution of the system (4.1)can be expressed in the following form

$$\begin{aligned} x(t) = x(t;\varphi) &+ \int_0^t (t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)\left[\int_{-h}^{0} d_\tau C(t,\tau)u(t+\tau)\right]ds \\ &+ \int_0^t (t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)\left(\int_0^\tau \sigma(\theta,x(\theta),x(\theta-h))\,d\omega(\theta)\right)ds \\ &+ \int_0^t (t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)f\big(s,x(s),(x-h)\big)\,ds. \end{aligned}$$

Using the well-known result of unsymmetric Fuubini theorem [19] and change of order of integration to the last term, we have

$$x(t) = x(t;\varphi) + \int_{-h}^{0} dC_\tau \left[\int_{\tau}^{0} \big(t-(s-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(t-(s-\tau)\big)C(s-\tau,\tau)\psi(s)\,ds\right]$$
$$+\int_0^t dC_\tau\left[\int_{-h}^{0}\big(t-(s-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(t-(s-\tau)\big)C_t(s-\tau,\tau)\right]u(s)ds$$
$$+\int_0^t (t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)\left(\int_0^\tau \sigma(\theta,x(\theta),x(\theta-h))\,d\omega(\theta)\right)ds$$
$$+\int_0^t (t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)\left[\int_{-h}^{0} d_\tau C(t,\tau)u(t+\tau)\right]ds$$
$$+\int_0^t (t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)f\big(s,x(s),(x-h)\big)\,ds, \qquad (4.2)$$

where

$$C_t(s,\tau) = \begin{cases} C(s,\tau), & s \le t \\ 0, & s > t \end{cases}$$

and $dC\tau$ denotes the integration of Lebesgue Stieltjes sense with respect to the variable τ in the function $C(t,\tau)$.

Define the control function

$$u(t) = H^*(T,s)W^{-1}\Bigg[x_1 - x(T,\varphi)$$
$$-\int_{-h}^{0} dC_\tau\left[\int_{\tau}^{0}\big(T-(s-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(T-(s-\tau)\big)C(s-\tau,\tau)\psi(s)ds\right]$$
$$-\int_0^t (t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)\int_0^\tau \sigma(\theta,x(\theta),x(\theta-h))d\omega(\theta)ds\Bigg]$$
$$-\int_0^t (t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)f\big(s,x(s),(x-h)\big)\,ds.$$

We assume the following hypotheses.

(H1) Let $h > 0$ be given. For functions $u : [-h,T] \to R^m$ and $t \in J$,we use thesymbol u_t to denote the function on $[-h,0]$, defined by $u_t(s) = u(t+s)$ for$s \in [-h,0)$.

(H2) $C(t,\tau)$ is an $n \times m$ dimensional matrix continuous in $t$for fixed $\tau$and is of bounded variation in τon $[-h,0]$ for each $t \in J$ and continuous from left in τ on the interval $(-h,0)$.

(H3) The functionsf,σ, and $g$are continuous and satisfies the usual linear growth condition, that is, there exists a constants $L > 0, M > 0$,
(i) $\|f(t,z,v)\|^2 \le L(1+\|z\|^2+\|v\|^2)$
(ii) $\|\sigma(t,z,v)\|^2 \le M(1+\|z\|^2+\|v\|^2)$
for all $t \in J$,and all $x \in R^n, u \in R^m$.

Theorem 2: Suppose that the hypothesis **(H1)–(H3)** are satisfied then the nonlinear system (4.1) is relatively controllable on J.Provided that the following hold$d = max\{m_4,m_5\}$, $c = max\{m_6,m_7\}$.

Proof: Define the operator $\Lambda: \mathcal{B} \to \mathcal{B}$by $\Lambda(z,v) = (x,u)$,with

$$u(t) = H^*(T,s)W^{-1}\left[x_1 - x(T,\varphi) - N(T) - \int_0^T (T-s)^{\alpha-1}\Phi_{\alpha,\alpha}(T-s)\int_0^\tau \sigma(\theta,x(\theta),x(\theta-h))d\omega(\theta)ds\right]$$
$$-\int_0^T (T-s)^{\alpha-1}\Phi_{\alpha,\alpha}(T-s)f\big(s,x(s),x(s-h)\big)ds,$$

and $x(t)$ is defined in (4.2). Then, it is easy to establish the following estimates

$$E\|u(t)\|^2 = E\Bigg\|H^*(T,s)W^{-1}\Bigg[x_1 - x(T,\varphi) - \int_{-h}^{0} dC_\tau\left[\int_\tau^0 \big(T-(s-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(T-(s-\tau)\big)C(s-\tau,\tau)\psi(s)\,ds\right] - \int_0^T (T-s)^{\alpha-1}\Phi_{\alpha,\alpha}(T-s)\int_0^\tau \sigma(\theta,x(\theta),x(\theta-h))d\omega(\theta)ds\Bigg] - \int_0^T (T-s)^{\alpha-1}\Phi_{\alpha,\alpha}(T-s)f(s,x(s),x(s-h))ds\Bigg\|^2,$$

$$= 6\|H^*(T,s)\|^2\|W^{-1}\|^2\Bigg[\|x_1\|^2 + \|x(T,\varphi)\|^2 + \left\|\int_{-h}^{0} dC_\tau\left[\int_\tau^0 \big(T-(s-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(T-(s-\tau)\big)C(s-\tau,\tau)\psi(s)\,ds\right]\right\|^2\Bigg] + \left(T^2 m_2 L + \frac{T^{2\alpha+1}}{\alpha^2}m_1 M_\sigma M\right)\left(1 + \sup_{t\in J}E\|z(t)\|^2 + \sup_{t\in J}E\|v(t)\|^2\right),$$

$$\le \frac{m_4}{4m_3} + \frac{m_6}{4m_3},$$

$$\le \frac{1}{4m_3}[d+c] = r_1.$$

$$E\|x(t)\|^2 \le 6\|x(t;\varphi)\|^2 + 6E\left\|\int_{-h}^{0} dC_\tau\left[\int_\tau^0 \big(t-(s-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(t-(s-\tau)\big)C(s-\tau,\tau)\psi(s)\,ds\right]\right\|^2 + 6E\left\|\int_0^t\left[\int_{-h}^0 \big(t-(s-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(t-(s-\tau)\big)d_\tau C_t(s-\tau,\tau)\right]u(s)ds\right\|^2 + 6E\left\|\int_0^t (t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)\left(\int_0^\tau \sigma\big(\theta,x(\theta),x(\theta-h)\big)\,d\omega(\theta)\right)ds\right\|^2 + 6E\left\|\int_0^t (t-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t-s)f\big(s,x(s),x(s-h)\big)ds\right\|^2,$$

$$\le 6E\Bigg[\|x(t;\varphi)\|^2 + T^2\|H^*(T,t)\|^2\left[\frac{1}{4m_3}(d+c)\right] + \left\|\int_{-h}^{0} dC_\tau\left[\int_\tau^0 \big(t-(s-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(t-(s-\tau)\big)C(s-\tau,\tau)\psi(s)\,ds\right]\right\|^2 + \left(T^2 m_2 L + \frac{T^{2\alpha+1}}{\alpha^2}m_1 M_\sigma M\right)\left(1 + \sup_{t\in J}E\|z(t)\|^2 + \sup_{t\in J}E\|v(t)\|^2\right)\Bigg],$$

$$\le \frac{m_5}{4} + \frac{1}{4}(d+c) + \frac{m_7}{4},$$

$$\le \frac{1}{2}[d+c] = r_2.$$

Therefore, $E\|u(t)\|^2 \le r_1$, for all $t \in J$, which gives $E\|x\|^2 \le r_2$. Thus, we have proved that, if $\boldsymbol{\mathcal{B}}(r') = \{(z,v) \in \boldsymbol{\mathcal{B}}: E\|z\|^2 \le r \text{ and } E\|v\|^2 \le r'\}$',then Λ maps $\boldsymbol{\mathcal{B}}(r')$ into itself.

Now let us take $t_1, t_2 \in J$with $t_1 < t_2$, and for all $(x,u) \in \boldsymbol{\mathcal{B}}(r)$,we need to show that $\Lambda[\boldsymbol{\mathcal{B}}(r)]$ is equicontinuous for all $r > 0$. To prove the result, compute

$$
\begin{aligned}
&E\|u(t_1)-u(t_2)\|^2 \\
&= \mathrm{E}\Bigg\|\mathrm{H}^*(\mathrm{T},\mathrm{t}_1)\mathrm{W}^{-1}\Bigg[\mathrm{x}_1-\mathrm{x}(\mathrm{T},\varphi) \\
&-\int_{-h}^{0}\mathrm{dC}_\tau\left[\int_\tau^0\big(\mathrm{T}-(\mathrm{s}-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(\mathrm{T}-(\mathrm{s}-\tau)\big)\mathrm{C}(\mathrm{s}-\tau,\tau)\psi(\mathrm{s})\,\mathrm{ds}\right] \\
&-\int_0^{\mathrm{T}}(\mathrm{T}-\mathrm{s})^{\alpha-1}\Phi_{\alpha,\alpha}(\mathrm{T}-\mathrm{s})\int_0^\tau\sigma(\theta,\mathrm{x}(\theta),\mathrm{x}(\theta-\mathrm{h}))\mathrm{d}\omega(\theta)\mathrm{ds}\Bigg] \\
&-\int_0^{\mathrm{T}}(\mathrm{T}-\mathrm{s})^{\alpha-1}\Phi_{\alpha,\alpha}(\mathrm{T}-\mathrm{s})\mathrm{f}(\mathrm{s},\mathrm{x}(\mathrm{s}),\mathrm{x}(\mathrm{s}-\mathrm{h}))\mathrm{ds}\Bigg\|^2 \\
&-\mathrm{E}\Bigg\|\mathrm{H}^*(\mathrm{T},\mathrm{t}_2)\mathrm{W}^{-1}\Bigg[\mathrm{x}_1-\mathrm{x}(\mathrm{T},\varphi) \\
&-\int_{-h}^{0}\mathrm{dC}_\tau\left[\int_\tau^0\big(\mathrm{T}-(\mathrm{s}-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(\mathrm{T}-(\mathrm{s}-\tau)\big)\mathrm{C}(\mathrm{s}-\tau,\tau)\psi(\mathrm{s})\,\mathrm{ds}\right] \\
&-\int_0^{\mathrm{T}}(\mathrm{T}-\mathrm{s})^{\alpha-1}\Phi_{\alpha,\alpha}(\mathrm{T}-\mathrm{s})\int_0^\tau\sigma\big(\theta,\mathrm{x}(\theta),\mathrm{x}(\theta-\mathrm{h})\big)\mathrm{d}\omega(\theta)\mathrm{ds}\Bigg] \\
&-\int_0^{\mathrm{T}}(\mathrm{T}-\mathrm{s})^{\alpha-1}\Phi_{\alpha,\alpha}(\mathrm{T}-\mathrm{s})\mathrm{f}\big(\mathrm{s},\mathrm{x}(\mathrm{s}),\mathrm{x}(\mathrm{s}-\mathrm{h})\big)\mathrm{ds}\Bigg\|^2,
\end{aligned}
$$

$$
\begin{aligned}
&= 6\|H^*(T,t_1)-H^*(T,t_2)\|^2\|W^{-1}\|^2\Bigg[\|x_1\|^2+\|x(T,\varphi)\|^2 \\
&+\Bigg\|\int_{-h}^{0}\mathrm{dC}_\tau\left[\int_\tau^0\big(\mathrm{T}-(\mathrm{s}-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(\mathrm{T}-(\mathrm{s}-\tau)\big)\mathrm{C}(\mathrm{s}-\tau,\tau)\psi(\mathrm{s})\,\mathrm{ds}\right]\Bigg\|^2\Bigg] \\
&+\left(T^2m_2L+\frac{T^{2\alpha+1}}{\alpha^2}m_1M_\sigma M\right)\left(1+\sup_{t\in J}E\|z(t)\|^2+\sup_{t\in J}E\|v(t)\|^2\right), \qquad (4.3)
\end{aligned}
$$

and

$$
\begin{aligned}
&E\|x(t_1)-x(t_2)\|^2 \\
&= E\Bigg\|\mathrm{x}(\mathrm{t}_1,\varphi) \\
&+\int_{-h}^{0}\mathrm{dC}_\tau\left[\int_\tau^0\big(\mathrm{t}_1-(\mathrm{s}-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(\mathrm{t}_1-(\mathrm{s}-\tau)\big)\mathrm{C}(\mathrm{s}-\tau,\tau)\psi(\mathrm{s})\,\mathrm{ds}\right] \\
&+\int_0^{\mathrm{t}_1}\left[\int_{-h}^{0}\big(\mathrm{t}_1-(\mathrm{s}-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(\mathrm{t}_1-(\mathrm{s}-\tau)\big)\mathrm{d}_\tau\mathrm{C}_{\mathrm{t}_1}(\mathrm{s}-\tau,\tau)\right]\mathrm{u}(\mathrm{s})\mathrm{ds} \\
&+\int_0^{\mathrm{t}_1}(\mathrm{t}_1-\mathrm{s})^{\alpha-1}\Phi_{\alpha,\alpha}(\mathrm{t}_1-\mathrm{s})\int_0^\tau\sigma\big(\theta,\mathrm{x}(\theta),\mathrm{x}(\theta-\mathrm{h})\big)\mathrm{d}\omega(\theta)\mathrm{ds} \\
&+\int_0^{\mathrm{t}_1}(\mathrm{t}_1-\mathrm{s})^{\alpha-1}\Phi_{\alpha,\alpha}(\mathrm{t}_1-\mathrm{s})\mathrm{f}(\mathrm{s},\mathrm{x}(\mathrm{s}),\mathrm{x}(\mathrm{s}-\mathrm{h}))\mathrm{ds}-\mathrm{x}(\mathrm{t}_2,\varphi) \\
&-\int_{-h}^{0}\mathrm{dC}_\tau\left[\int_\tau^0\big(\mathrm{t}_2-(\mathrm{s}-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(\mathrm{t}_2-(\mathrm{s}-\tau)\big)\mathrm{C}(\mathrm{s}-\tau,\tau)\psi(\mathrm{s})\,\mathrm{ds}\right] \\
&-\int_0^{\mathrm{t}_2}\left[\int_{-h}^{0}\big(\mathrm{t}_2-(\mathrm{s}-\tau)\big)^{\alpha-1}\Phi_{\alpha,\alpha}\big(\mathrm{t}_2-(\mathrm{s}-\tau)\big)\mathrm{d}_\tau\mathrm{C}_{\mathrm{t}_2}(\mathrm{s}-\tau,\tau)\right]\mathrm{u}(\mathrm{s})\mathrm{ds} \\
&-\int_0^{\mathrm{t}_2}(\mathrm{t}_2-\mathrm{s})^{\alpha-1}\Phi_{\alpha,\alpha}(\mathrm{t}_2-\mathrm{s})\int_0^\tau\sigma\big(\theta,\mathrm{x}(\theta),\mathrm{x}(\theta-\mathrm{h})\big)\mathrm{d}\omega(\theta)\mathrm{ds} \\
&-\int_0^{\mathrm{t}_2}(\mathrm{t}_2-\mathrm{s})^{\alpha-1}\Phi_{\alpha,\alpha}(\mathrm{t}_2-\mathrm{s})\mathrm{f}(\mathrm{s},\mathrm{x}(\mathrm{s}),\mathrm{x}(\mathrm{s}-\mathrm{h}))\mathrm{ds}\Bigg\|^2
\end{aligned}
$$

$$\leq 10\Bigg\{\|x(t_1;\varphi)-x(t_2;\varphi)\|^2$$
$$+\left\|\int_{-h}^{0} dC_\tau\left[\int_\tau^0\left[\left(t_1-(s-\tau)\right)^{\alpha-1}\Phi_{\alpha,\alpha}\left(t_1-(s-\tau)\right)\right.\right.\right.$$
$$\left.\left.\left.-\left(t_2-(s-\tau)\right)^{\alpha-1}\Phi_{\alpha,\alpha}\left(t_2-(s-\tau)\right)\right]C(s-\tau,\tau)\psi(s)\,ds\right]\right\|^2$$
$$+(t_2-t_1)\int_{t_1}^{t_2}\|H(t_2,s)\|^2E\|u(s)\|^2ds$$
$$+t_1\int_0^{t_1}\|H(t_1,s)-H(t_2,s)\|^2E\|u(s)\|^2ds$$
$$+(t_2-t_1)\int_{t_1}^{t_2}\left\|(t_2-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t_2-s)\right\|^2L(1+E\|z(s)\|^2+E\|v(s)\|)^2ds$$
$$+t_1\int_0^{t_1}\left\|(t_1-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t_1-s)-(t_2-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t_2-s)\right\|^2L(1+E\|z(s)\|^2+E\|v(s)\|)^2ds$$
$$+(t_2-t_1)\int_{t_1}^{t_2}(t_2-s)^{2(\alpha-1)}\left\|(t_2-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t_2-s)\right\|^2TM_\sigma M(1+E\|z(s)\|^2+E\|v(s)\|)^2ds$$
$$+t_1\int_0^{t_1}\left\|(t_1-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t_1-s)-(t_2-s)^{\alpha-1}\Phi_{\alpha,\alpha}(t_2-s)\right\|^2TM_\sigma M(1+E\|z(s)\|^2$$
$$+E\|v(s)\|)^2ds\Bigg\}. \qquad (4.4)$$

Moreover, for all $(x,u)\in\boldsymbol{\mathcal{B}}(r)$,

$$E\|u(t)\|^2\leq 6\|H^*(T,s)\|^2\|W^{-1}\|^2\Bigg[\|x_1\|^2+\|x(T,\varphi)\|^2$$
$$+\left\|\int_{-h}^{0}dC_\tau\left[\int_\tau^0\left(T-(s-\tau)\right)^{\alpha-1}\Phi_{\alpha,\alpha}\left(T-(s-\tau)\right)C(s-\tau,\tau)\psi(s)\,ds\right]\right\|^2\Bigg]$$
$$+T\int_0^T\left\|(T-s)\Phi_{\alpha,\alpha}(T-s)\right\|^2E\left\|f\left(s,x(s),x(x-h)\right)\right\|^2$$
$$+TM_\sigma\int_0^T(T-s)^{2(\sigma-1)}\left[\int_0^s E\|\sigma(\theta),x(\theta),x(\theta-h)\|^2\right]\left\|(T-s)^{\alpha-1}\Phi_{\alpha,\alpha}(T-s)\right\|^2ds.$$

Thus the right sides of the Equations (4.3) and (4.4) are independent of $(x,u)\in\boldsymbol{\mathcal{B}}(r)$ and tend to zero as $t_1\to t_2$. Hence $\Lambda[\boldsymbol{\mathcal{B}}(r)]$ is equicontinuous for all $r>0$ and by the regularity assumptions on f *and* $\sigma$are continuous, and hence it is completely continuous by the application of Arzela -Ascoli's theorem. Since $\boldsymbol{\mathcal{B}}(r')$ is closed, bounded and convex,the Schauder fixed point theorem guarantees that Λ has a fixed point $(z,v)\in\boldsymbol{\mathcal{B}}(r)$ such that $\Lambda(z,v)=(z,v)\equiv(x,u)$. Hence $x(t)$ is the solution of the system (4.1), andit is easy to verify that $x(T)=x_1$. Further the control function $u(t)\in\mathcal{U}_{ad}$ steers the system (4.1) from initial complete state $y(0)$ to x_1 on J. Hence the system (4.1) is relatively controllable on J.

5.Example

Consider the nonlinear fractional stochastic delay dynamical system with distributed delays in control of the form

$$cD^{\frac{3}{2}}x(t)=Ax(t)+Bx(t-1)+\int_{-1}^{0}d_\tau C(t,\tau)u(t+\tau)+\sigma\left(t,x(t),x(t-1)\right)+f\left(t,x(t),x(t-1)\right),\ (5.1)$$

where $t\in[0,1],\alpha=\frac{3}{2},h=1,x(t)=\varphi(t),x'(t)=\varphi'(t)\in R^2$,

$$A\begin{pmatrix}0&1\\-1&0\end{pmatrix},\ B=\begin{pmatrix}0&0\\1&0\end{pmatrix}C(t,\tau)=\begin{pmatrix}e^\tau\cos t&e^\tau\sin t\\-e^\tau\sin t&e^\tau\cos t\end{pmatrix},\sigma\left(t,x(t),x(t-1)\right)$$
$$=\begin{pmatrix}(2t^2+1)x_1(t)e^{-t}&0\\0&x_2(t-1)e^{-t}\end{pmatrix},$$

$$f(t,x(t),x(t-1)) = \begin{pmatrix} \dfrac{10x_1(t)}{1+x_1^2(t)+x_2^2(t-1)} \\ \dfrac{x_2(t-1)}{1+x_2^2(t-1)+x_2^2(t)} \end{pmatrix}.$$

The Mittag-Leffler matrix function of the systems is given by

$$E_{\frac{3}{2}}\left(At^{\frac{3}{2}}\right) = \begin{pmatrix} \sum_{j=0}^{\infty} \dfrac{(-1)^j t^{3j}}{\Gamma[1+3j]} & \sum_{j=0}^{\infty} \dfrac{(-1)^j t^{\frac{3}{2}(2j+1)}}{\Gamma\left[1+\frac{3}{2}(2j+1)\right]} \\ -\sum_{j=0}^{\infty} \dfrac{(-1)^j t^{\frac{3}{2}(2j+1)}}{\Gamma\left[1+\frac{3}{2}(2j+1)\right]} & \sum_{j=0}^{\infty} \dfrac{(-1)^j t^{3j}}{\Gamma[1+3j]} \end{pmatrix},$$

$$E_{\frac{3}{2},\frac{3}{2}}\left(A\left(T-(s-\tau)\right)^{\frac{3}{2}}\right) = \begin{pmatrix} \sum_{j=0}^{\infty} \dfrac{(-1)^j \left(T-(s-\tau)\right)^{3j}}{\Gamma\left[\frac{3}{2}(1+2j)\right]} & \sum_{j=0}^{\infty} \dfrac{(-1)^j \left(T-(s-\tau)\right)^{\frac{3}{2}(2j+1)}}{\Gamma[1+3(j+1)]} \\ -\sum_{j=0}^{\infty} \dfrac{(-1)^j \left(T-(s-\tau)\right)^{\frac{3}{2}(2j+1)}}{\Gamma[1+3(j+1)]} & \sum_{j=0}^{\infty} \dfrac{(-1)^j \left(T-(s-\tau)\right)^{3j}}{\Gamma\left[\frac{3}{2}(1+2j)\right]} \end{pmatrix},$$

and

$$\left(T-(s-\tau)\right)^{\frac{1}{2}} E_{\frac{3}{2},\frac{3}{2}}\left(A\left(T-(s-\tau)\right)^{\frac{3}{2}}\right) = \begin{pmatrix} \cos_{\frac{3}{2}}(t) & \sin_{\frac{3}{2}}(t) \\ -\sin_{\frac{3}{2}}(t) & \cos_{\frac{3}{2}}(t) \end{pmatrix},$$

where $\cos_{\frac{3}{2}}(t)$ and $\sin_{\frac{3}{2}}(t)$ are given by

$$\cos_{\frac{3}{2}}(t) = \sum_{j=0}^{\infty} \frac{(-1)^j \left(T-(s-\tau)\right)^{\frac{3}{2}(2j+1)-1}}{\Gamma\left[\frac{3}{2}(1+2j)\right]},$$

$$\sin_{\frac{3}{2}}(t) = \sum_{j=0}^{\infty} \frac{(-1)^j \left(T-(s-\tau)\right)^{3(j+1)-1}}{\Gamma[3(1+j)]}.$$

Also,

$$H(T,s) = \int_{-1}^{0} \left(\mathrm{T}-(\mathrm{s}-\tau)\right)^{\frac{1}{2}} \Phi_{\frac{3}{2},\frac{3}{2}}\left(\mathrm{A}\left(\mathrm{T}-(\mathrm{s}-\tau)\right)^{\frac{3}{2}}\right) \mathrm{d}_\tau \mathrm{C}_1(\mathrm{s}-\tau,\tau),$$
$$= \begin{pmatrix} \mathrm{p(s)} & \mathrm{q(s)} \\ -\mathrm{q(s)} & \mathrm{p(s)} \end{pmatrix},$$

where

$$p(s) = \int_{-1}^{0} e^{\tau} \left[\cos_{\frac{3}{2}}\left(1-(\mathrm{s}-\tau)\right)\cos(s-\tau) - \sin_{\frac{3}{2}}\left(1-(\mathrm{s}-\tau)\right)\sin(s-\tau)\right] d\tau,$$

$$q(s) = \int_{-1}^{0} e^{\tau} \left[\cos_{\frac{3}{2}}\left(1-(\mathrm{s}-\tau)\right)\sin(s-\tau) + \sin_{\frac{3}{2}}\left(1-(\mathrm{s}-\tau)\right)\cos(s-\tau)\right] d\tau.$$

Thus, from simple matrix calculation we can obtain the controllability matrix

$$W(0,1) = \int_0^1 H(1,s)H^*(1,s)ds,$$
$$= \int_0^1 [p^2(s)+q^2(s)] \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} ds,$$

which is positive definite. Further it is easy to verity the hypothesis $(H1)-(H3)$ hold, hence all the assumptions in Theorem 2 are satisfied. Thus the fractional order system (5.1) is relatively controllable on [0,1].

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Operation κ - Closed Graphs

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ABSTRACT: *In this paper, the concept of κ - closed graphs on operation κ is developed. Some characterizations and properties related to κ - closed graphs are analyzed.*

Keywords:

2. Introduction

Levine [10] introduced the concept of semi-open sets in 1963. Following this notion, Arya and Nour [1] in 1990 introduced generalized semi closed sets. In 1979, Kasahara [8] developed the concept of α-closed graphs of a function with the help of certain operation of topology. Later in 1983, Jankovic [6] investigated the functions with α-closed graphs. Jayashree and Sivakamasundari [7] defined the Operation approaches on gs-open sets in topological spaces in 2018.
Following the work of Operation approaches on gs-open sets in topological spaces, κ - closed graphs on operation and its properties are studied in this article.

3. Preliminaries

Definition 3.1 [7]
Let (X,τ) be a topological space. A mapping $\kappa : GSO(X,\tau) \to P(X)$ from the family of generalized semi open sets $GSO(X,\tau)$ to the power set of X such that $V \subseteq V^{\kappa}$ for every $V \in GSO(X,\tau)$ where V^{κ} denotes the value of V under the operation κ.

Definition 3.2 [7]
A subset A of a space (X,τ) will be called a ***κ-open set*** of (X,τ) if for each $x \in A$, there exists a gs-open set U of x and $U^{\kappa} \subseteq A$.
$\boldsymbol{\kappa O(X,\tau)}$will denote the set of all κ-open sets.

Definition 3.3 [7]
A κ-operation $\kappa : GSO(X,\tau) \to P(X)$ is called $\boldsymbol{regular_{\kappa}}$**operation** given $x \in X$ and for each pair of gs-open sets A and B of x, there exists a gs-open set C of x such that $A^{K} \cap B^{K} \supseteq C^{K}$.

Definition 3.4 [7]
A topological space (X,τ) is called ***κ- regular*** if for given $x \in X$ and each gs-open set U of x,, there exists a gs-open set V of x such that $V^{\kappa} \subseteq U$.

Definition 3.5 [7]
A subset A of a topological space (X,τ) is called ***κ- closed*** whenever $X - A$ is κ-open.

Definition 3.6 [7]
Let κ be an operation on $GSO(X,\tau)$. A point $x \in X$ is said to be ***κ- closure point*** of the set A if $U^{\kappa} \cap A \neq \phi$ for each gs-openset U of x. $\boldsymbol{gs\ Cl_{\kappa}(A)} = \{x \in X / U^{\kappa} \cap A \neq \phi,\ \forall gsopen\ set\ U \text{ of } x\}$

Definition 3.7 [7]
Let κ be an operation on $GSO(X,\tau)$. Then $\boldsymbol{gs_{\kappa}Cl(A)}$ is defined as the intersection of all κ-closed sets containing A.
$\boldsymbol{gs_{\kappa}Cl(A)} = \cap \{F \subseteq X / \ A \subseteq F \text{ and } X \backslash F \in \kappa O(X,\tau)\}$.

Definition 3.8 [7]
An operation κ on $GSO(X,\tau)$ is said to be $\boldsymbol{open_\kappa}$operation if for every gs-open set U of $x \in X$, there exists a κ-open set V such that $x \in V$ and $V \subset U^\kappa$.

Definition 3.9 [4]
A subset A of a topological space (X,τ) is called **locally closed if** $A = U \cap F$ where U is open and F is closed in (X,τ).

Definition 3.10 [14]
A subset A of (X,τ) is called **regular open** if $A = int(cl(A))$.

Remark 3.11 [3]
A subset A of a space (X,τ) is called a $\boldsymbol{T_b}$**- space** if every gs-closed set in X is closed.

4. $\boldsymbol{\kappa}$ - Closed Graphs

Definition 4.1
The **Graph $\boldsymbol{G(f)}$ of a function** $f: X \to Y$ is a $\boldsymbol{\kappa}$**-closed** if for each $(x,y) \in (X \times Y) - G(f)$ there exist a open set U and a gs-open set V containing x and y respectively such that $(U \times V^\kappa) \cap G(f) = \emptyset$.

Lemma 4.2
A function $f: X \to Y$ has a κ-closed graph if and only if for each $(x,y) \in (X \times Y) - G(f)$ there exist gs- open sets U and V containing x and y respectively such that $f(U) \cap V^\kappa = \emptyset$.

Proof
Let $f: X \to Y$ has a κ-closed graph. Let $(x,y) \in (X \times Y) - G(f)$and an open set U and a gs-open set V are containing x and y respectively. Suppose $f(U)\,) \cap V^\kappa \neq \emptyset$. Then there exists $z \in f(U)$ and $z \in V^\kappa$. $z \in f(U)$ implies there exist $x \in U$ such that $z = f(x)$. Consider the order pair $(x,z) \in U \times V^\kappa$. Since $z = f(x)$, $(x,z) \in G(f)$. Therefore we get $(U \times V^\kappa) \cap G(f) \neq \emptyset$ which contradicts the definition of κ-closed graph. Retracing the steps we get the converse.

Definition 4.3
A subset A of X is $\boldsymbol{\kappa}$**-compact** if for every κ-open cover $\mathbb{C}$ of A there exists a finite subfamily $\{U_1, U_2, \ldots\ldots, U_n\}$ of $\mathbb{C}$ such that $A \subseteq \bigcup_{i=1}^{n} U_i^{\kappa}$.

Remark 4.4
If κ is the identity operation on τ, then κ compact set is compact.

Theorem 4.5
Let (X,τ) and (Y,τ') be two topological spaces. Let κ be a regular κ-operation τ' and let $f: X \to Y$ be a function with a κ-closed graph. If A is a compact subset of X, then $f(A)$ is a κ-closed subset of Y.

Proof
Let A be a comact subset of X. Suppose $f(A)$ is not κ-closed in Y. Then there exists $y \in gscl_\kappa\big(f(A)\big) - f(A)$. That is $y \notin f(A)$. Implies $y \neq f(x)$ for any $x \in A$. Since f has a κ-closed graph, By the above lemma we get for each $x \in A$, there exist an open set U_x and a gs-open set V_x containing x and y such that $f(U_x) \cap V_x^{\kappa} = \emptyset$. Consider $\mathcal{A}$=$\{U_x / x \in A\}$ which is a open cover of A. Since A is compact there is a finite subcollection $\{U_{x_1}, U_{x_2}, \ldots\ldots, U_{x_n}\}$ of $\mathcal{A}$ such that $A \subseteq \bigcup_{i=1}^{n} U_{x_i}$. Consider the corresponding V_{x_i}, $i = 1$ to n in Y $[V_{x_1}, V_{x_2}, \ldots\ldots, V_{x_n}]$. This is a collection of gs-open sets. Consider $\{V_{x_1}^{\kappa}, V_{x_2}^{\kappa}, \ldots\ldots, V_{x_n}^{\kappa}\}$. By regularity of κ-operation there exists a gs-open set V such that $V^\kappa \subseteq V_{x_1}^{\kappa} \cap V_{x_2}^{\kappa}, \cap \ldots\ldots, \cap V_{x_n}^{\kappa}$. We Know $A \subseteq \bigcup U_{x_i}$. Therefore $f(A) \subseteq f(\bigcup U_{x_i}) \subseteq \bigcup f(U_{x_i})$. Take $f(A) \cap V^\kappa \subseteq \bigcup f(U_{x_i}) \cap V^\kappa \subseteq \bigcup(f(U_{x_i}) \cap V^\kappa) \subseteq \bigcup(f(U_{x_i}) \cap V_{x_i}^{\kappa} = \emptyset$. Implies $y \notin gscl_\kappa(A)$. Which contradicts the definition of κ-closure. Therefore $gscl_\kappa\big(f(A)\big) = f(A)$ implies $f(A)$ is κ-closed.

Theorem 4.6
Let $f: X \to Y$ be a function with a κ-closed graph. If B is a κ-compact subset of Y. Then $f^{-1}(B)$ is a closed subset of X.

Proof
The proof is similar to the proof of the Theorem 4.5.

Definition 4.7
A function $f: X \to Y$ is a **locally** $\boldsymbol{\kappa}$**-closed** if for open set U of x there is gs-open set V of x such that $V \subseteq U$ and $f(V)$ is κ-closed in Y.

Remark 4.8
If κ is the identity operation on τ, where X is g-$T_{1/2}$ then locally κ-closedness of a function coincides with locally closedness.

Definition 4.9
A function $f: X \to Y$ is called $\boldsymbol{\kappa}$**-closed mapping** if f maps closed subsets of X onto κ-closed subsets of Y.

Definition 4.10
A function $f: X \to Y$ is called **almost κ-closed** subsets of Y if f maps regular closed subsets of X onto κ-closed subsets of Y.

Remark 4.11
Every κ-closed function is an almost κ-closed function.

Proof
Let f be a κ-closed function. To prove f is almost κ-closed function, consider a regularly closed set U in X. Since every regularly closed set is closed and since f is a κ-closed function, $f(U)$ is κ-closed in (Y, σ). Hence f is an almost κ-closed function.

Remark 4.12
It is obvious that the class of κ-closed functions is contained in the class of almost κ-closed functions.

Remark 4.13
Also, if the domain of an almost κ-closed functions is regular, then the function is κ-closed.

Lemma 4.14
If a function $f: X \to Y$ is locally κ-closed and has closed point inverses, then f has an κ-closed graph.

Proof
Consider graph $g(f) = \{(x, f(x))/\, x \in X\}$. Let $(x, y) \in X \times Y - G(f)$. Then $(x, y) \notin G(f)$ implies $y \neq f(x)$ which shows $x \notin f^{-1}(y)$. Since f has closed point inverses, f^{-1} of every singleton is closed. Implies $f^{-1}(y)$ is closed in (X, τ). Since $x \notin f^{-1}(y)$ there exists an open set U conatining x such that $U \cap f^{-1}(y) = \emptyset$. $x \in U$ and by the definition of locally κ-closed function there exists a gs-open set V containing x and contained in U such that $f(V)$ is κ-closed in Y. Since $U \cap f^{-1}(y) = \emptyset$ we get $V \cap f^{-1}(y) = \emptyset$ implies $y \notin f(V)$ which is a κ-closed set. Then there exists a gs- open set W containing x such that $W^{\kappa} \cap f(V) = \emptyset$. Let V_0 be a gs-open set containing x such that $V_0 \subseteq V$. Implies $f(V_0) \cap W^{\kappa} = \emptyset$. By lemma 2, we have f has a κ-closed graph.
The following theorem is an immediate consequence of the above lemma and by the remark 4.13.

Theorem 4.15
If a function $f: X \to Y$ is almost κ-closed with closed point inverses and its domain X is a regular space, then f has a κ-closed graph.
The following theorem is a proof that the converse of the Theorem 4.5 is true if X is locally compact and regular.

Theorem 4.16
Let (X, τ) and (Y, σ) be two topological spaces. Let κ be a regular κ-operation on σ and if $f: X \to Y$ is a function where X is locally compact and regular with T_b space, then the following conditions are equivalent.

(a) f maps compact sets onto κ-closed sets and has closed point inverses.
(b) f is locally κ-closed and has closed point inverses.
(c) f has a κ-closed graph.

Proof
$(a) \Rightarrow (b)$
Let U be an open set containing a point x in X. Since X is locally compact and regular, X has a compact basis of each point and hence, there is a compact open set V of x such that $V \subset U$. Since every open set is gs-open, V is gs-open. By (a) f maps the compact set V to a κ-closed set. Therefore, $f(V)$ is a κ-closed and (b) is verified.
$(b) \Rightarrow (c)$
By Lemma 4.14, (b) implies (c).
$(a) \Rightarrow (c)$
If f has a κ-closed graph, then Theorem 4.5 gives that f maps compact sets onto κ-closed sets. Since singleton is κ-compact, Theorem 6 establishes that f has closed point inverses.

Remark 4.17
If κ is the identity operation of σ then Theorem 4.16 becomes Theorem 3.11 of [2]

Lemma 4.18
If a function $f: X \to Y$ is almost κ-closed and has ϑ closed point inverses, then f *has a* κ-closed graph.

Proof
Let $(x, y) \in (X \times Y) - G(f)$. Then $x \notin f^{-1}(y)$ and since $f^{-1}(y)$ is ϑ closed, there is a open set U of x such that $cl(U) \cap f^{-1}(y) = \emptyset$. Since $cl(U)$ is regularly closed, the almost κ-closedness of f implies that $f(cl(U))$ is κ-closed in Y. Since $y \notin f(cl(U))$, there is a gs-open set V of Y such that $f\big(cl(U)\big) \cap V^{\kappa} = \emptyset$ implies $f(U) \cap V^{\kappa} = \emptyset$ By Lemma 4.2 it follows that f has an κ -closed graph.

Remark 4.19

Since ϑ-closure and closure coincides for subsets of a regular space Theorem 4.15 follows from Lemma 4.18.

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UNIFICATION OF REGULAR STAR OPEN SETS

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ABSTRACT: *In this article, a new open set which has places between a* $\boldsymbol{\delta}$*-open set and open set called as a* $\eta*$ *-open set is initiated and its basic properties are procured. Also the concepts of* $\eta*$ *- cluster point,* $\eta*$ *-adherent point and a* $\eta*$ *-derived set are introduced and studied.*

Keywords: *$\eta*$-open set, $\eta*$ -closed set, $\eta*$ - cluster point, $\eta*$ -adherent point.*

1.Introduction

In 1937, Stone [8] introduced regular open sets and used it to define the semi-regularization of a topological space. In 1968,Velicko [9] proposed δ-open sets which are stronger than open sets and proved that the collection of $\boldsymbol{\delta}$-open sets denoted by τ_δ formed a coarser topology on (X,τ).Levine [3] has brought generalized closed sets in 1970. Dunham [2] has established a generalized closure using Levine's generalized closed sets. In 2016,Annalakshmi [7] has instituted regular*-open sets. In this article, a new open set which has places between a $\boldsymbol{\delta}$-Open Set and open set called as a $\eta*$ -open set is initiated and its basic properties are procured. Also the concepts of $\eta*$ - cluster point, $\eta*$ -adherent point and a $\eta*$ -derived set are introduced and studied.

2.PRELIMINARIES

Throughout this paper, (Y,ζ) will always denote topological space on which no separation axioms are assumed, unless explicitly stated. If A is a subset of the space (Y,ζ), Cl(A) and Int(A) denote the closure and interior of A respectively.

Definition 2a:

If A is a subset of a space (Y,ζ),
(i) The generalized closure of A [2] is defined as the intersection of all g-closed sets in X containing A and is denoted by Cl*(A).
(ii) The generalized interior of A [2] is defined as the union of all g-open sets in X contained in A and is denoted by Int*(A).

Definition 2b:

A subset A of a topological space (Y,ζ) is said to be
(i) regular open[8] if A=Int(Cl(A)).
(ii) regular star open(or r* -open)[7] if A=Int(Cl*(A))
(iii) α-open [6] if A ⊆Int(Cl(Int(A))).
(iv) semi-open [4] if A ⊆Cl(Int(A)).
(v) pre-open [5] if A ⊆Int(Cl(A)).
(vi) semi-pre-open [1] if A ⊆Cl(Int(Cl(A))).
(vii) δ-open set [9] if it is the union of regular open sets.

The complements of the above mentioned sets are called regular closed,regular*- closed,α-closed,semi-closed,pre-closed,semi-pre-closed,δ-closed sets respectively.
The intersection of all regular closed (resp.,regular*-closed, α-closed,semi-closed,pre-closed,semi-pre-closed, δ-closed) subsets of (Y,ζ) containing A is called the regular closure(resp.,regular*-closure, α-closure,semi-closure,pre-closure,semi-pre-closure, δ-closure) of A and are denoted by rCl(A)(resp.,r*-Cl(A), α-Cl(A),sCl(A),pCl(A),spCl(A), δCl(A)).

Definition 2c:

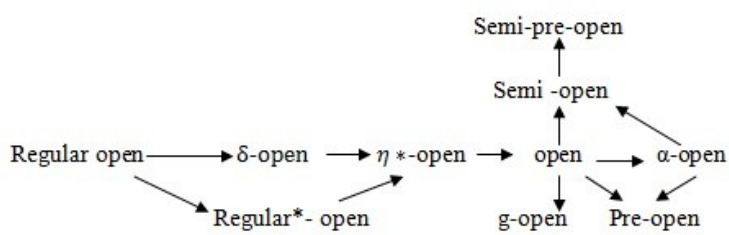
A subset A of a topological space (Y,ζ) is said to be
(i) generalized closed [3](briefly g-closed) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in Y.
(ii) generalized open [3](briefly g-open) if $X - A$ is g-closed in Y.

Definition 2d:

If A is a subset of a space (Y,ζ) ,
(i) The δ- interior of A is defined as the union of all δ-open sets of Y contained in A. It is denoted by δint(A).
(ii) The δ-closure of A is defined as the intersection of all δ-open sets of Y contained in A. It is denoted by δCl(A).

3.$\eta*$-Open sets

$\eta*$-open Set-Definition 3a:
A subset D of a topological space (Y,ζ) is called $\eta*$-open set if it is a union of regular*-open sets (r*-open sets).
We denote the set of all $\eta*$-open sets in (Y,ζ) by $\eta*$-OS(Y,ζ) or $\eta*$-OS(Y)
Example 3b:
Let Y={p,q,r}, ζ={ϕ, Y, {p},{q},{p,q}}. Then r*-open sets ={ϕ, Y, {p},{q}}, $\eta*$-OS(Y) ={ϕ, Y, {p},{q},{p,q}}.
Theorem 3c:
Every δ-open set is a $\eta*$-open set but not conversely.
Proof
Let D be a δ-open set.Take $x \in D$,then $x \in \cup B_\mu$,where B_μ is regular open.Since every regular open is regular*-open by Theorem 3.12 of Annalakshmi [3].Then $x \in C_\mu \subseteq \cup C_\mu$,where C_μ is regular*-open.Therefore D is $\eta*$-open.Hence every δ-open is $\eta*$-open.
Counter Example 3d:
Let Y={p,q,r}, ζ={ϕ, Y,{p}}. Then the subset {p} is a $\eta*$-open set but it is not a δ-open set in(Y,ζ).
Theorem 3e:
Every $\eta*$-open set is a open set but not conversely.
Proof
Let D be a $\eta*$-open set.Take $x \in D$,then $x \in \cup B_\mu$,where B_μ is r*-open.Since every regular*- open is open by Theorem 3.15 of Annalakshmi [3].Then $x \in C_\mu$, where C_μ is open. Therefore D is open.Hence every $\eta*$-open is open.
Counter Example 3f:
Let Y={p,q,r}, ζ={ϕ, Y, {p,q}}. Then the subset {p,q} is a open set but it is not a $\eta*$-open set in (Y,ζ).
Theorem 3g:
Every r*-open set is a $\eta*$-open set but not conversely.
Proof
Follows from $\eta*$-open set-Definition 3a.
Counter Example 3h:
Let Y={p,q,r}, ζ ={ϕ, Y, {p},{q},{p,q}}. Then the subset {p,q} is a $\eta*$-open set but it is not a r*-open set in (Y,ζ).
From the above theorems we get the following implication diagram



Theorem 3i:
(a) In any topological space (Y,ζ),Y and ϕ are $\eta*$-open sets.
(b) Any arbitrary union of $\eta*$-open sets are $\eta*$-open sets.
(c) The finite intersection of $\eta*$-open sets are $\eta*$-open sets.
Result 3j:

$\eta*$-OS(Y) forms a topology which is finer than τ and coarser than τ_{δ}.

$\eta*$-Interior of a subset-Definition 3k:

A subset D of a topological space (Y,ζ) is called $\eta*$-Interior of D is the union of all $\eta*$-open sets of Y contained in D.We denote the symbol by $\eta*$-Int(D).

Result 3l:

(i) $\eta*$-Int(D) is $\eta*$-open.

(ii) $\eta*$-Int(D) is the maximum $\eta*$-open set contained in D.

Theorem 3m:

D is $\eta*$-open iff D = $\eta*$-Int(D).

Proof

($\Longrightarrow$) Let D be a $\eta*$-open set.$\eta*$-Int(D) is the union of all $\eta*$-open sets of Y contained in D.That is $\eta*$-Int(D) is the maximum $\eta*$-open set contained in D.Therefore $\eta*$-Int(D) = D.

($\Longleftarrow$) Let D = $\eta*$-Int(D).Since $\eta*$-Int(D) is the union of all $\eta*$-open sets of Y contained in D.Then it is $\eta*$-open.

Properties 3n:

In any topological space (Y,ζ), if D and E are subsets of Y then we get the following :

a. $\eta*$-Int(ϕ) = ϕ,
b. $\eta*$-Int(Y) = Y,
c. $\eta*$-Int(D) $\subseteq$D,
d. D $\subseteq$ E $\Rightarrow$ $\eta*$-Int(D) $\subseteq$ $\eta*$-Int(E),
e. Int($\eta*$-Int(D)) $\subseteq$ Int(D),
f. δInt(D) $\subseteq$ $\eta*$-Int(D) $\subseteq$ Int(D) $\subseteq$ D,
g. $\eta*$-Int(D$\cup$E) = $\eta*$-Int(D) $\cup$ $\eta*$-Int(E),
h. $\eta*$-Int(D$\cap$E) = $\eta*$-Int(D) $\cap$ $\eta*$-Int(E).

4. $\eta*$-closed sets

$\eta*$-closed set-Definition 4a:

The complement of a $\eta*$-open set is called a $\eta*$-closed set.We denote $\eta*$-closed sets in (Y,ζ) by $\eta*$-CS(Y,ζ) or $\zeta*$-CS(Y).

Theorem 4b:

Every $\eta*$-closed set is closed but not conversely.

Proof

Let D be a $\eta*$closed set in (Y,ζ),then X$-$D is $\eta*$open.By **Theorem 3e** , X$-$D is open implies that D is closed.Hence every $\eta*$-closed set is closed.

Counter Example 4c:

Let Y={p,q,r}, ζ={ϕ, Y,{p},{p,q}}. Then the subset {r} is closed but not $\eta*$-closed in (Y,ζ).

Theorem 4d:

Every **δ**-closed is $\eta*$-closed but not conversely.

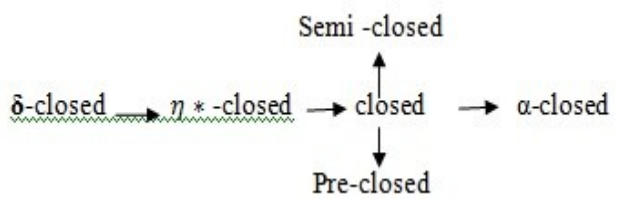
Proof

Let D be a **δ**-closed set in (Y,ζ),then X$-$D is **δ**-open.By **Theorem 3c**, X$-$D is $\eta*$-open implies that D is $\eta*$-closed.Hence every **δ**-closed is $\eta*$-closed.

Counter Example 4e:

Let Y={p,q,r}, ζ={ϕ, Y, {p}}. Then the subset {q,r} is a $\eta*-$-closed set but it is not a **δ**-closed set.

From the above theorems we get the following diagrammatic implications:



$\eta*$ closure of a subset-Definition 4f:

The intersection of all $\eta*$-closed sets of Y containing D is called as the $\eta*$-closure of D and denoted by $\eta*$-Cl(D).

Note 4g:

$\eta*$-Cl(D) is $\eta*$-closed.

$\eta*$-adherent point-Definition 4h:

Let D $\subseteq$Y. An element x$\in$Y is called $\eta*$-adherent point of D if every $\eta*$-open set in Y containing x intersects D.

$\eta*$-cluster point-Definition 4i:

Let D be a subset of the topological space (Y,ζ).A point x$\in$Y is called a $\eta*$-cluster point of D if for every $\eta*$-open set V containing x intersects D in a point different from x.

$\eta*$-Derived set-Definition 4j:

We denote the set of all $\eta*$-cluster points of D by $\eta*$-D(D).

Properties 4k:

In any topological space (Y,ζ),we get the following result hold:

a. $\eta*$-Cl($\emptyset$) = $\emptyset$
b. $\eta*$-Cl(Y) = Y
c. D$\subseteq$ $\eta*$-Cl(D)
d. D$\subseteq$ E$\Rightarrow$ $\eta*$-Cl(D) $\subseteq$ $\eta*$ -Cl(E),if D and E are subsets of Y.
e. D$\subseteq$ Cl(D) $\subseteq$ $\eta*$-Cl(D) $\subseteq$ δCl(D)
f. $\eta*$-Cl(D $\cup$ E) = $\eta*$-Cl(D) $\cup$ $\eta*$-Cl(E),if D and E are subsets of Y,
g. $\eta*$-Cl(D $\cap$ E) = $\eta*$-Cl(D) $\cap$ $\eta*$-Cl(E),if D and E are subsets of Y,
h. Cl(D) $\subseteq$ Cl($\eta*$-Cl(D)).

5.Conclusion

After studying $\eta*$-open sets and its properties,we have proposed to introduce J- closed sets,J * - closed sets and J ** - closed sets in topological spaces.

A subset D of a topological space (Y,ζ) is said to be J**-closed set** if
Cl(D) $\subseteq$ M whenever D $\subseteq$ M, M is $\eta*$-open in (Y,ζ).

A subset D of a topological space (Y,ζ) is said to be J * **-closed set** if
$\eta*$-Cl(D) $\subseteq$ M whenever D $\subseteq$ M, M is open in (Y,ζ).

A subset D of a topological space (Y,ζ) is said to be J ** **-closed set** if
$\eta*$-Cl(D) $\subseteq$ M whenever D $\subseteq$ M, M is $\eta*$-open in (Y,ζ).

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Some Forms of NVGP-Irresolute Mappings

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ABSTRACT: *In this paper we consider the weak and strong forms of neutrosophic vague topological continuous maps via the concept of NVGPCS which is called as NVGP ap-irresolute maps and NVGP ap-closed maps and also used them to obtain the characterization of* $\mathrm{NV_{gp}T_{1/2}}$ *space.*

Keywords: *Neutrosophic Vague topology, NVGP ap-irresolute maps, NVGP ap-closed maps,* $\mathrm{NV_{gp}T_{1/2}}$ *space.*

1.Introduction:

The concept of fuzzy sets was introduced by Zadeh [10] in 1965. Using this fuzzy sets intuitionistic fuzzy sets was introduced by Atanassov [1] in 1986. In 1970, the generalized closed sets was initiated the studied by Levine [5]. Fuzzy topology was introduced by C.L.Chang [3] in 1967. The theory of vague sets was first proposed and developed as an extension of fuzzy set theory by Gau and Buehrer [4]. Then, Smarandache[9] introduces the neutrosophic elements T, I, F which represent the membership, indeterminacy, and non-membership values respectively, where]−0,1+[is that the non-standard unit interval in 1998. Shawkat Alkhazaleh[8] in 2015 introduced and constructed the concept of neutrosophic vague set. With this neutrosophic vague set in 2017 Mary Margaret and Trinita Pricilla[6],[7] developed the concept of neutrosophic vague topology and neutrosophic vague continuous and irresolute mapping. In 2000, M.Caldas[2] defined and studied weak and strong forms of irresolute maps in general topology.

In this paper we extend the same concept in neutrosophic vague topological spaces. We introduce the concept of irresoluteness called neutrosophic vague generalized pre ap-irresolute maps and neutrosophic vague generalized pre ap-closed maps and some of their basic properties.

2. Preliminaries

Definition 2.1:[8] A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X written as $A_{NV} = \left\{\left\langle x; \hat{T}_{A_{NV}}(x); \hat{I}_{A_{NV}}(x); \hat{F}_{A_{NV}}(x)\right\rangle; x \in X\right\}$, whose truth membership, indeterminacy membership and false membership functions is defined as:

$$\hat{T}_{A_{NV}}(x) = \left[T^-, T^+\right], \hat{I}_{A_{NV}}(x) = \left[I^-, I^+\right], \hat{F}_{A_{NV}}(x) = \left[F^-, F^+\right]$$

where,

1) $T^+ = 1 - F^-$
2) $F^+ = 1 - T^-$ and
3) $^-0 \le T^- + I^- + F^- \le 2^+$.

Definition 2.2:[8] Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$, $\hat{T}_{A_{NV}}(u_i) \le \hat{T}_{B_{NV}}(u_i)$; $\hat{I}_{A_{NV}}(u_i) \ge \hat{I}_{B_{NV}}(u_i)$; $\hat{F}_{A_{NV}}(u_i) \ge \hat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} is included by B_{NV}, denoted by $A_{NV} \subseteq B_{NV}$, where $1 \le i \le n$.

Definition 2.3:[8] The complement of NVS A_{NV} is denoted by A^c_{NV} and is defined by

$$\hat{T}^c_{A_{NV}}(x) = \left[1 - T^+, 1 - T^-\right], \hat{I}^c_{A_{NV}}(x) = \left[1 - I^+, 1 - I^-\right], \hat{F}^c_{A_{NV}}(x) = \left[1 - F^+, 1 - F^-\right].$$

Definition 2.4:[8] Let A_{NV} be NVS of the universe U where $\forall u_i \in U$, $\hat{T}_{A_{NV}}(x) = [1,1]$; $\hat{I}_{A_{NV}}(x) = [0,0]$; $\hat{F}_{A_{NV}}(x) = [0,0]$. Then A_{NV} is called a unit NVS(1_{NV} in short), where $1 \le i \le n$.

Definition 2.5:[8] Let A_{NV} be NVS of the universe U where $\forall u_i \in U$, $\hat{T}_{A_{NV}}(x)=[0,0]$; $\hat{I}_{A_{NV}}(x)=[1,1]$; $\hat{F}_{A_{NV}}(x)=[1,1]$. Then A_{NV} is called a zero NVS(0_{NV} in short), where $1 \le i \le n$.

Definition 2.6:[8] The union of two NVSs A_{NV} and B_{NV} is NVS C_{NV}, written as $C_{NV} = A_{NV} \cup B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by,

$$\hat{T}_{C_{NV}}(x)=\left[\max\left(T^-_{A_{NV_x}}, T^-_{B_{NV_x}}\right), \max\left(T^+_{A_{NV_x}}, T^+_{B_{NV_x}}\right)\right]$$
$$\hat{I}_{C_{NV}}(x)=\left[\min\left(I^-_{A_{NV_x}}, I^-_{B_{NV_x}}\right), \min\left(I^+_{A_{NV_x}}, I^+_{B_{NV_x}}\right)\right]$$
$$\hat{F}_{C_{NV}}(x)=\left[\min\left(F^-_{A_{NV_x}}, F^-_{B_{NV_x}}\right), \min\left(F^+_{A_{NV_x}}, F^+_{B_{NV_x}}\right)\right].$$

Definition 2.7:[8] The intersection of two NVSs A_{NV} and B_{NV} is NVS C_{NV}, written as $C_{NV} = A_{NV} \cap B_{NV}$, whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by,

$$\hat{T}_{C_{NV}}(x)=\left[\min\left(T^-_{A_{NV_x}}, T^-_{B_{NV_x}}\right), \min\left(T^+_{A_{NV_x}}, T^+_{B_{NV_x}}\right)\right]$$
$$\hat{I}_{C_{NV}}(x)=\left[\max\left(I^-_{A_{NV_x}}, I^-_{B_{NV_x}}\right), \max\left(I^+_{A_{NV_x}}, I^+_{B_{NV_x}}\right)\right]$$
$$\hat{F}_{C_{NV}}(x)=\left[\max\left(F^-_{A_{NV_x}}, F^-_{B_{NV_x}}\right), \max\left(F^+_{A_{NV_x}}, F^+_{B_{NV_x}}\right)\right].$$

Definition 2.8:[8] Let A_{NV} and B_{NV} be two NVSs of the universe U. If $\forall u_i \in U$, $\hat{T}_{A_{NV}}(u_i)=\hat{T}_{B_{NV}}(u_i)$; $\hat{I}_{A_{NV}}(u_i)=\hat{I}_{B_{NV}}(u_i)$; $\hat{F}_{A_{NV}}(u_i)=\hat{F}_{B_{NV}}(u_i)$, then the NVS A_{NV} and B_{NV}, are called equal, where $1 \le i \le n$.

Definition 2.9:[6] A neutrosophic vague topology (NVT in short) on X is a family τ of neutrosophic vague sets (NVS in short) in X satisfying the following axioms:

- $0_{NV}, 1_{NV} \in \tau$
- $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
- $\cup G_i \in \tau, \forall \{G_i : i \in J\} \subseteq \tau$

In this case the pair (X,τ) is called a neutrosophic vague topological space (NVTS in short) and any NVS in τ is known as a neutrosophic vague open set (NVOS in short) in X. The complement A^c of NVOS in a NVTS (X,τ) is called neutrosophic vague closed set (NVCS in short) in X.

Definition 2.10:[6] A NVS $A=\left\{\left\langle x,\left[\hat{T}_A,\hat{I}_A,\hat{F}_A\right]\right\rangle\right\}$ in NVTS (X,τ) is said to be

i) *Neutrosophic Vague pre- closed set* (NVPCS in short) if $NVcl(NV\operatorname{int}(A)) \subseteq A$,

ii) *Neutrosophic Vague pre-open set* (NVPOS in short) if $A \subseteq NV\operatorname{int}(NVcl(A))$,

Definition 2.11:[6] Let A be NVS of a NVTS (X,τ). Then the neutrosophic vague pre interior of A ($NV\mathrm{p}\operatorname{int}(A)$ in short) and neutrosophic vague pre closure of A ($NV\mathrm{p}cl(A)$ in short) are defined by

- $NV\mathrm{p}\operatorname{int}(A) = \cup\{G / G \text{ is a NVPOS in } X \text{ and } G \subseteq A\}$,
- $NV\mathrm{p}cl(A) = \cap\{K / K \text{ is a NVPCS in } X \text{ and } A \subseteq K\}$.

Definition 2.12:[6] A NVS A of a NVTS (X,τ) is said to be neutrosophic vague generalized closed set (NVGCS in short) if $NVcl(A) \subseteq U$ whenever $A \subseteq U$ and U is NVOS in X.

Definition 2.13:[6] A NVS A is said to be neutrosophic vague generalized pre-closed set (NVGPCS in short) in (X,τ) if $NV\text{p}cl(A)\subseteq U$ whenever $A\subseteq U$ and U is NVOS in X.

Definition 2.14:[6] A NVTS (X,τ) is said to be neutrosophic vague ${}_{gp}T_{1/2}$ space (NV ${}_{gp}T_{1/2}$ in short) if every NVGPCS in X is NVCS in X.

Definition 2.15:[7] A map $f:(X,\tau)\to(Y,\sigma)$ is said to be neutrosophic vague generalized pre-continuous (NVGP continuous in short) mapping if $f^{-1}(A)$ is NVGPCS in (X,τ) for every NVCS A of (Y,σ).

Definition 2.16:[7] A map $f:(X,\tau)\to(Y,\sigma)$ is said to be neutrosophic vague generalized pre irresolute (NVGP irresolute in short) mapping if $f^{-1}(A)$ is NVGPCS in (X,τ) for every NVGPCS A in (Y,σ).

3. NVGP contra irresolute, NVGP ap-irresolute, NVGP ap-closed maps:

Definition 3.1: A mapping $\phi:(U,\tau)\to(V,\sigma)$ is said to be NVGP irresolute if $\phi^{-1}(B)$ is NVGPOS in U for every NVGPOS in *V*.

Definition 3.2: A mapping $\phi:(U,\tau)\to(V,\sigma)$ is said to be NVGP contra irresolute if $\phi^{-1}(B)$ is NVGPCS in *U* for every NVGPOS in *V*.

Definition 3.3: A mapping $\phi:(U,\tau)\to(V,\sigma)$ is said to be neutrosophic vague generalized pre approximately irresolute (NVGP ap-irresolute) if $NV\text{p}cl(A)\subseteq\phi^{-1}(B)$, whenever *B* is a NVOS in *V*, *A* is NVGPCS in *U* and $A\subseteq\phi^{-1}(B)$.

Definition 3.4: A mapping $\phi:(U,\tau)\to(V,\sigma)$ is said to be neutrosophic vague generalized pre approximately closed (NVGP ap-closed) if $\phi(B)\subseteq NV\text{p}\,\text{int}(A)$, whenever *A* is a NVGPOS in *V*, *B* is NVCS in *U* and $\phi(B)\subseteq A$.

Theorem 3.5: NV irresolute mapping is NVGP ap-irresolute mapping.

Proof: Let ϕ be NV irresolute mapping and *B* is NVOS in *V*, *A* is NVGPCS in *U* such that $A\subseteq\phi^{-1}(B)$. By our assumption $\phi^{-1}(B)$ is NVOS in *U*. Then by the definition of NVGPCS we have $NV\text{p}cl(A)\subseteq\phi^{-1}(B)$. Hence ϕ is NVGP ap-irresolute mapping.

Theorem 3.6: A mapping $\phi:(U,\tau)\to(V,\sigma)$ is:

i) NVGP ap-irresolute if $\phi^{-1}(A)$ is NVOS in U for every NVOS *A* in *V*.

ii) NVGP ap-closed if $\phi(B)$ is NVCS in V for every NVCS *B* in *U*.

Proof:

i) Let A be NVOS in *V*, $\phi^{-1}(A)$ is NVOS in U and *B* be NVGPCS in U such that $B\subseteq\phi^{-1}(A)$. By the definition of NVGPCS in U we have, $NV\text{p}cl(B)\subseteq\phi^{-1}(A)$ whenever $B\subseteq\phi^{-1}(A)$ and $\phi^{-1}(A)$ is NVOS in *U*. Thus ϕ is NVGP ap-irresolute.

ii) Let B be NVCS in *X*, $\phi(B)$ is NVCS in V and A be NVGPOS in V such that $\phi(B)\subseteq A$. By the definition of NVGPOS in *V* we have, $\phi(B)\subseteq NV\text{p}\,\text{int}(A)$ whenever $\phi(B)\subseteq A$ and $\phi(B)$ is NVCS in *V*. Thus ϕ is NVGP ap-closed.

Theorem 3.7: Let $\phi:(U,\tau)\to(V,\sigma)$ be a mapping from a NVTS (U,τ) into a NVTS (V,σ).

i) If NVOS and NVCS of (U,τ) coincide, then ϕ is NVGP ap-irresolute if and only if $\phi^{-1}(B)$ is NVOS in *U* for every NVOS *B* in *V*.

iii) If NVOS and NVCS of (U,τ) coincide, then ϕ is NVGP ap-closed if and only if $\phi(B)$ is NVCS in *V* for every NVCS *B* in *U*.

Proof:

i) Assume the ϕ is NVGP ap-irresolute, then by the definition we have, $NV\text{p}cl(A) \subseteq \phi^{-1}(B)$, whenever B is a NVOS in *V*, *A* is NVGPCS in *U* and $A \subseteq \phi^{-1}(B)$. Since *A* is NVGPCS in *U*, then by the definition it follows that $\phi^{-1}(B)$ is NVOS in *U*.
The converse part follows from the theorem 3.6.

ii) It is similar to (i).

Corollary 3.8: Let $\phi:(U,\tau)\to(V,\sigma)$ be a mapping from a NVTS (U,τ) into a NVTS (V,σ).

i) If NVOS and NVCS of (U,τ) coincide, then ϕ is NVGP ap-irresolute if and only if ϕ is NV irresolute mapping.

ii) If NVOS and NVCS of (Y,σ) coincide, then ϕ is NVGP ap-closed if and only if ϕ is NV irresolute mapping.

Proof: It is obvious.

Remark 3.9: NVGP contra irresoluteness and NVGP irresoluteness are independent of each other it is shown from the following example.

Example 3.10: Let $U=\{a,b\}$, $V=\{x,y\}$ and

$$A=\left\{u,\frac{a}{\langle[0.7,0.8];[0.1,0.4];[0.2,0.3]\rangle},\frac{b}{\langle[0.6,0.9];[0.1,0.3];[0.1,0.4]\rangle}\right\},$$

$$B=\left\{v,\frac{x}{\langle[0.5,0.9];[0.6,0.8];[0.1,0.5]\rangle},\frac{y}{\langle[0.7,0.8];[0.5,0.7];[0.2,0.3]\rangle}\right\}.$$

Then $\tau=\{0_{NV},A,1_{NV}\}$ and $\sigma=\{0_{NV},B,1_{NV}\}$ are NVTs on *U* and *V* respectively. Define a mapping $\phi:(U,\tau)\to(V,\sigma)$ by $\phi(a)=x$ and $\phi(b)=y$. Clearly ϕ is NVGP contra irresolute mapping but not NVGP irresolute mapping, since B is NVGPOS in *V*, but $\phi^{-1}(B)$ is not NVGPOS in *U*.

Example 3.11: Let $U=\{a,b\}$, $V=\{x,y\}$ and

$$A=\left\{u,\frac{a}{\langle[0.2,0.5];[0.4,0.8];[0.5,0.8]\rangle},\frac{b}{\langle[0.3,0.6];[0.5,0.9];[0.4,0.7]\rangle}\right\},$$

$$B=\left\{v,\frac{x}{\langle[0.1,0.3];[0.6,0.9];[0.7,0.9]\rangle},\frac{y}{\langle[0.2,0.5];[0.7,0.9];[0.5,0.8]\rangle}\right\}.$$

Then $\tau=\{0_{NV},A,1_{NV}\}$ and $\sigma=\{0_{NV},B,1_{NV}\}$ are NVTs on *U* and *V* respectively. Define a mapping $\phi:(U,\tau)\to(V,\sigma)$ by $\phi(a)=x$ and $\phi(b)=y$. Clearly ϕ is NVGP irresolute mapping but not NVGP contra irresolute mapping, since B is NVGPOS in *V*, but $\phi^{-1}(B)$ is not NVGPCS in *U*.

Theorem 3.12: Let $\phi:(U,\tau)\to(V,\sigma)$ be a mapping from NVTS (U,τ) into a NVTS (V,σ). Then the following conditions are equivalent:

i) ϕ is NVGP contra irresolute.

ii) The inverse image of each NVGPOS in V is NVGPCS in *U*.

Proof: It is obvious.

Definition 3.13: A mapping $\phi:(U,\tau)\to(V,\sigma)$ is said to be NVGP perfectly irresolute if the inverse image of every NVGPOS in V is NVGP clopen in *U*.

Theorem 3.14: Every NVGP perfectly irresolute mapping is NVGP contra irresolute mapping.

Proof: Let ϕ is NVGP perfectly irresolute mapping and let B be NVGPOS in *V*. Then by our assumption $\phi^{-1}(B)$ is NVGP clopen in *U*. Thus $\phi^{-1}(B)$ is NVGPCS in *U*. Hence NVGP contra irresolute mapping.

The converse of the above theorem is not true in general as seen from the following example.

Example 3.15: Let $U=\{a,b\}, V=\{x,y\}$ and

$$A=\left\{u,\frac{a}{\langle[0.4,0.5];[0.1,0.3];[0.5,0.6]\rangle},\frac{b}{\langle[0.3,0.6];[0.2,0.4];[0.4,0.7]\rangle}\right\},$$

$$B=\left\{v,\frac{x}{\langle[0.6,0.9];[0.1,0.2];[0.1,0.4]\rangle},\frac{y}{\langle[0.7,0.8];[0.2,0.3];[0.2,0.3]\rangle}\right\}.$$

Then $\tau=\{0_{NV},A,1_{NV}\}$ and $\sigma=\{0_{NV},B,1_{NV}\}$ are NVTs on U and V respectively. Define a mapping $\phi:(U,\tau)\to(V,\sigma)$ by $\phi(a)=x$ and $\phi(b)=y$. Clearly ϕ is NVGP contra irresolute mapping but not NVGP perfectly irresolute mapping, since B is NVGPOS in V, but $\phi^{-1}(B)$ is NVGPCS and not NVGPOS in U.

Theorem 3.16: Every NVGP perfectly irresolute mapping is NVGP irresolute mapping.

Proof: It is obvious.

The converse of the above theorem is not true in general as seen from the following example.

Example 3.17: Let $U=\{a,b\}, V=\{x,y\}$ and

$$A=\left\{u,\frac{a}{\langle[0.3,0.5];[0.6,0.8];[0.5,0.7]\rangle},\frac{b}{\langle[0.2,0.6];[0.5,0.8];[0.4,0.8]\rangle}\right\},$$

$$B=\left\{v,\frac{x}{\langle[0.2,0.3];[0.7,0.9];[0.7,0.8]\rangle},\frac{y}{\langle[0.1,0.4];[0.6,0.8];[0.6,0.9]\rangle}\right\}.$$

Then $\tau=\{0_{NV},A,1_{NV}\}$ and $\sigma=\{0_{NV},B,1_{NV}\}$ are NVTs on U and V respectively. Define a mapping $\phi:(U,\tau)\to(V,\sigma)$ by $\phi(a)=x$ and $\phi(b)=y$. Clearly ϕ is NVGP irresolute mapping but not NVGP perfectly irresolute mapping, since B is NVGPOS in V, but $\phi^{-1}(B)$ is NVGPOS and not NVGPCS in U.

Theorem 3.18: Let $\phi:(U,\tau)\to(V,\sigma)$ be a mapping from NVTS (U,τ) into a NVTS (V,σ). Then the following conditions are equivalent:

i) ϕ is NVGP perfectly irresolute.

ii) ϕ is NVGP contra irresolute and NVGP irresolute.

Proof: $(\text{i})\Rightarrow(\text{ii})$: Let ϕ be NVGP perfectly irresolute mapping. Let A be NVGPOS in V. By our assumption $\phi^{-1}(A)$ is NVGP clopen in U. Thus $\phi^{-1}(A)$ is both NVGPOS and NVGPCS in U. Hence ϕ is NVGP contra irresolute and NVGP irresolute.

$(\text{ii})\Rightarrow(\text{i})$: Let ϕ be both NVGP contra irresolute and NVGP irresolute and let A be NVGPOS in V. By our assumption $\phi^{-1}(A)$ is both NVGPOS and NVGPCS in U that is $\phi^{-1}(A)$ is NVGP clopen in U. Hence ϕ is NVGP perfectly irresolute.

Theorem 3.19: Let $\phi:(U,\tau)\to(V,\sigma)$ and $\eta:(V,\sigma)\to(W,\mu)$ be two mapping. Then,

i) $\eta\circ\phi$ is NVGP ap-irresolute, if ϕ is NVGP ap-irresolute and η is NV irresolute.

ii) $\eta\circ\phi$ is NVGP ap-closed, if ϕ is NV closed and η is NVGP ap-closed.

Proof: i) Assume that A is NVGPCS in U and B is NVOS in W for which $A\subseteq(\eta\circ\phi)^{-1}(B)$. Since η is NV irresolute $\eta^{-1}(B)$ is NVOS in V. Since ϕ is NVGP ap-irresolute $NV\text{pcl}(A)\subseteq\phi^{-1}[\eta^{-1}(B)]=(\eta\circ\phi)^{-1}(B)$. This proves that $\eta\circ\phi$ is NVGP ap-irresolute mapping.

ii) Assume that A is NVGPOS in W and B is NVCS in U for which $(\eta\circ\phi)(B)\subseteq A$. Then $\phi(B)$ is NVCS in V, as ϕ is NV closed. Since η is NVGP ap-closed $\eta(\phi(B))=(\eta\circ\phi)(B)\subseteq NV\text{pint}(A)$. This implies that $\eta\circ\phi$ is NVGP ap-closed.

Theorem 3.20: Let $\phi:(U,\tau)\to(V,\sigma)$ be a mapping from a NVTS (U,τ) into a NVTS (V,σ) and $\eta:(V,\sigma)\to(W,\mu)$ be a mapping from a NVTS (V,σ) into a NVTS (W,μ) respectively. Then,

i) $\eta\circ\phi$ is NVGP perfectly irresolute, if ϕ and η are NVGP perfectly irresolute.

ii) $\eta\circ\phi$ is NVGP contra irresolute, if ϕ is NVGP perfectly irresolute and η is NVGP contra irresolute.

iii) $\eta\circ\phi$ is NVGP irresolute, if ϕ is NVGP perfectly irresolute and η is NVGP contra irresolute.

iv) $\eta\circ\phi$ is NVGP irresolute, if ϕ is NVGP perfectly irresolute and η is NVGP irresolute.

v) $\eta\circ\phi$ is NVGP contra irresolute, if ϕ is NVGP perfectly irresolute and η is NVGP irresolute.

vi) $\eta\circ\phi$ is NVGP irresolute, if ϕ is NVGP contra irresolute and η is NVGP contra irresolute.

vii) $\eta\circ\phi$ is NVGP contra irresolute, if ϕ is NVGP contra irresolute and η is NVGP irresolute.

Proof: It follows from the definition.

Theorem 3.21: If $\phi:(U,\tau)\to(V,\sigma)$ is NVGP contra irresolute from a NVTS (U,τ) into a NVTS (V,σ), then ϕ is NVGP contra continuous mapping.

Proof: Let A be NVOS in *V*. Since every NVOS is NVGPOS, A is NVGPOS in *V*. Since ϕ is NVGP contra irresolute, then $\phi^{-1}(A)$ is NVGPCS in *U*. Hence ϕ is NVGP contra continuous mapping.

Example 3.22: Let $U=\{a,b\}$, $V=\{x,y\}$ and

$$A=\left\{u,\frac{a}{\langle[0.6,0.9];[0.3,0.5];[0.1,0.4]\rangle},\frac{b}{\langle[0.7,0.9];[0.4,0.7];[0.1,0.3]\rangle}\right\},$$

$$B=\left\{v,\frac{x}{\langle[0.8,0.9];[0.7,0.8];[0.1,0.2]\rangle},\frac{y}{\langle[0.4,0.8];[0.6,0.8];[0.2,0.6]\rangle}\right\}.$$

Then $\tau=\{0_{NV},A,1_{NV}\}$ and $\sigma=\{0_{NV},B,1_{NV}\}$ are NVTs on *U* and *V* respectively. Define a mapping $\phi:(U,\tau)\to(v,\sigma)$ by $\phi(a)=x$ and $\phi(b)=y$. Clearly ϕ is NVGP contra continuous mapping. But $C=\left\{v,\frac{x}{\langle[0.3,0.8];[0.4,0.8];[0.2,0.7]\rangle},\frac{y}{\langle[0.5,0.7];[0.6,0.8];[0.3,0.5]\rangle}\right\}$ is NVGPOS in *V* and $\phi^{-1}(C)$ is not NVGPCS in *U*. Hence ϕ is not NVGP contra irresolute mapping.

4. Characterization of $\mathbf{NV_{gp}T_{1/2}}$ space:

Theorem 4.1: Let (U,τ) be a NVTS. Then the following statements are equivalent.

i) (U,τ) is $\mathrm{NV_{gp}T_{1/2}}$ space.

ii) For every NVTS (V,σ) and every mapping $\phi:(U,\tau)\to(V,\sigma)$ is NVGP ap-irresolute mapping.

Proof: $(\mathrm{i})\Rightarrow(\mathrm{ii})$: Let *B* be NVGPCS in *U* and suppose that $B\subseteq\phi^{-1}(A)$ where *A* is NVOS in *V*. Since (U,τ) is $\mathrm{NV_{gp}T_{1/2}}$ space, then *B* is NVPCS in *U*, which implies $NV\mathrm{p}cl(B)=B$. Therefore $NV\mathrm{p}cl(B)\subseteq\phi^{-1}(A)$. Therefore ϕ is NVGP ap-irresolute mapping.

$(\mathrm{ii})\Rightarrow(\mathrm{i})$: Let *B* be NVGPCS in *U* and NVOS in *V*, then by the definition of NVGP ap-irresolute we have $B\subseteq\phi^{-1}(B)$, it follows that $NV\mathrm{p}cl(B)\subseteq\phi^{-1}(B)=B$. Hence *B* is NVPCS in *U*. Therefore *U* is $\mathrm{NV_{gp}T_{1/2}}$ space.

Theorem 4.2: Let (V,σ) be a NVTS. Then the following statements are equivalent.

i) (V,σ) is $\mathrm{NV_{gp}T_{1/2}}$ space.

ii) For every NVTS (U,τ) and every mapping $\phi:(U,\tau)\to(V,\sigma)$ is NVGP ap-closed mapping.

Proof: The proof is similar to theorem 4.1.

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TRANSIENT ANALYSIS OF M/M/c WITH VACATION AND BALKING

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ABSTRACT: *In this paper we analyze multiple server markovian queueing model with Balking and vacation, in which the arrival are allowed to join the queue in a poisson distribution and the service takes place according to an exponential distribution. There are finite numbers of c servers with different service rates and each arriving customer requires one server for its service. It is assumed that arriving customers balk with a fixed probability and whenever the system is empty the server goes for the vacation and returns to the system at the end of the vacation and starts providing service to the waiting customer. By using probability generating function along with the Bessel properties the explicit time dependant probabilities of system size is also obtained.*

Keywords: *Transient Analysis, Laplace transform, Vacation , Balking*

Introduction:

A queueing model is constructed so that queue lengths and waiting time can be predicted. The first paper on queuing theory was published in 1909 by A.K.Erlang. After him many authors have dealt with queueing models having different arrival and service rates. Parthasarathy [6] solved a time-dependant for the number in a single server queuing system with poission arrival and exponent service time. Krishna kumar and Arivudainambi [4] discussed the asymptotic behavior of the probability of the server being idle and obtained the steady state probabilities for the same. El-Shehawy S A and Ammar S I[2] in his paper presents the analysis of M/M/c queue with balking and reneging. Kumar and Sharma [5] studied a single as well as multi-server Markovian queuing systems with reneging, balking and retention of reneged customers.Dharmaraja and Rakesh Kumar [1] studied the markovian queuing system with heterogeneous servers and catastrophes, some special cases of the model are derived and the probability that the arriving customer finds the system busy and average number of server busy in the steady state are obtained.

Shyam sundar [8] in his paper deals with the erlangian queuing system with time dependant framework. Vijayashree and Janani [10]describes that server goes for vacation and return back to the system after N or more customers are found in the system further an explicit expressions for the time dependent system size probabilities using Laplace transform and generating function techniques are obtained. Shanmugasundaram and Chitra [7] deals with time dependant solution of M/M/c feedback queue under catastrophic effect. Jua-Chaun ke and Fu-Min Chan [3] deals with an *M/M/c* balking retrial queue with vacation,both single and multiple vacation policies are analysed. Sherif I Ammar [9] derived an explicit solution for M/M/1 queue where the server is allowed to take vacation whenever the system is empty for a random period of time. Based on the above literature available , we obtain the transient solution of M/M/c queueing model with vacation and balking.

Model description

Consider a multi server markovian queuing model in which customers are allowed to join the system according to the poisson distribution with parameter λ and service follows an exponential distribution with parameter μ . The first four equations follows with the initial condition that the severs in the system are in the vacation mode and the second set of equations follows with the condition that there are in the busy state. The vacation parameter with the parameter θ and balking denoted by P the server goes for a vacation if the system becomes empty and returns back if there are more than c customers waiting for service. The customers balk the system when there are c or more customers in the system.

By standard methods, the system of Kolmogorov differential difference equations governing the process is given by

$$P_{00}^{'}(t) = -\lambda P_{00}(t) + \mu P_{11}(t) \qquad n=0 \qquad (1)$$

$$P_{0n}^{'}(t) = -\lambda P_{0n-1}(t) - \lambda P_{0n}(t) \qquad 1 \le n < c \qquad (2)$$

$$P_{0n}^{'}(t) = -\lambda P P_{0n}(t) + \lambda P_{0n-1}(t) \qquad n=c \qquad (3)$$

$$P_{0n}^{'}(t) = -(\lambda P + \theta) P_{0n}(t) + \lambda P P_{0n-1}(t) \qquad n > c$$

(4) $P_{11}^{'}(t) = -(\lambda+\mu) P_{11}(t) + 2\mu P_{12}(t) \qquad n=1$

(5)

$$P_{1n}^{'}(t) = -(\lambda + n\mu) P_{1n}(t) + (n+1)\mu P_{1n+1}(t) + \lambda P_{1n-1} \qquad 2 \le n < c$$

(6)

$$P_{1n}^{'}(t) = -(\lambda + c\mu) P_{1n}(t) + \lambda P_{1n-1}(t) + c\mu P_{1n+1}(t) \qquad n=c$$

(7)

$$P_{1n}^{'}(t) = -(\lambda + c\mu) P_{1n}(t) + \lambda P P_{1n-1}(t) + c\mu P_{1n+1}(t) + \theta P_{0n}(t) \qquad n > c$$

(8)

To solve the above set of differential difference equation, we define

$$P(z,t) = q_c(t) + \sum_{n=1}^{\infty} P_{1n+c}^{'}(t) z^n \qquad and \qquad q_c(t) = \sum_{n=1}^{c} P_{1n}^{'}(t)$$

From equations (5) (6) and (7) we get,

$$\sum_{n=1}^{c} P_{1n}^{'}(t) = \quad -\lambda P P_{1c}(t) + c\mu P_{1c+1}(t) - \mu P_{11}(t) \qquad (9)$$

Now, multiplying equ.(8) by z^n and summing over the corresponding limit values

$$\frac{d[\sum_{n=1}^{\infty} P_{1c+n}(t) z^n]}{dt} = [-(\lambda P + c\mu) + \lambda P z + \frac{c\mu}{z}] \sum_{n=1}^{\infty} P_{1c+n}(t) z^n + \lambda P z P_{1c}(t) - c\mu P_{1c+1}(t) + \theta \sum_{n=1}^{\infty} P_{0c+n}(t) z^n \qquad (10)$$

Adding equs. (9) & (10) and using we get

$$\frac{\partial P(z,t)}{\partial t} = \left[-(\lambda z + c\mu) + \lambda P z + \frac{c\mu}{z} \right] (P(z,t) - q_c(t) + \lambda P(z-1) P_{1c}(t) + \theta \sum_{n=1}^{\infty} P_{0c+n}(t) z^n - \mu P_{11} \qquad (11)$$

By solving partial differential equ .(11) and there by using integrating factor, yields,

$$P(z,t) = \exp\left\{ \left[(\lambda P + c\mu) + \left(\lambda P z + \frac{c\mu}{z} \right) \right] t \right\} + \int_0^t \left\{ \lambda P(z-1) P_{1c}(u) - \left[\left(\lambda P z + \frac{c\mu}{z} \right) - (\lambda P + c\mu) \right] q_c(u) \right\}$$

$$\exp\left\{ \left[\left(\lambda P z + \frac{c\mu}{z} \right) - (\lambda P + c\mu) \right] (t-u) \right\} + \theta \int_0^t \sum_{n=1}^{\infty} P_{0c+n}(u) z^n$$

$$\exp\left\{ \left[\left(\lambda P z + \frac{c\mu}{z} \right) - (\lambda P + c\mu) \right] (t-u) \right\} - \mu \int_0^t P_{11}(u)$$

$$\exp\left\{ \left[\left(\lambda P z + \frac{c\mu}{z} \right) - (\lambda P + c\mu) \right] (t-u) \right\} du$$

By letting $\alpha = 2\sqrt{\lambda P c \mu}$ and $\beta = \sqrt{\frac{\lambda P}{c\mu}}$

And by applying modified Bessel function we get,

$$\exp\left\{\left(\lambda Pz+\frac{c\mu}{z}\right)t\right\}=\sum_{n=-\infty}^{\infty}(\beta z)^n I_n(\alpha t)$$

Comparing the coefficients of z^n we get and $\gamma = c\mu$,

$$\begin{aligned}P_{n+c}(t) &= \exp\{-(\lambda P+\gamma)t\}\beta^n I_n(\alpha t)\\ &+\lambda P\int_0^t P_{1c}(u)\exp\{-(\lambda P+\gamma)(t-u)\left[I_{n-1}(\alpha(t-u))\beta^{n-1}-I_n(\alpha(t-u))\beta^n\right]\}du\\ &-\int_0^t q_{1c}(u)\exp\{-(\lambda+\gamma)(t-u)\}\\ &*\left[\lambda P\beta^{n-1}I_{n-1}(\alpha(t-u))-(\lambda P+\gamma)\beta^n I_n(\alpha(t-u))+\gamma\beta^{n+1}I_{n+1}(\alpha(t-u))\right]\\ &-\mu\int_0^t P_{11}(u)\exp\{-(\lambda+\gamma)(t-u)\}\beta^n I_n(\alpha(t-u))du+\theta\int_0^t\sum_{m=1}^{\infty}P_{0c+m}(u)z^m\\ &\exp\{-(\lambda+\gamma)(t-u)\}(\beta z)^{n-m}I_{n-m}(\alpha(t-u))du \qquad (14)\end{aligned}$$

Comparing the terms free from z on either side , we get

$$\begin{aligned}q_c(t) &= \exp\{-(\lambda P+\gamma)t\}I_0(\alpha t)+\lambda P\int_0^t exp\{-(\lambda+\gamma)(t-u)\}P_{1c}(u)\left[\beta^{-1}I_1(\alpha(t-u))-I_0(\alpha(t-u))\right]du\\ &+\int_0^t\exp\{-(\lambda+\gamma)(t-u)\}q_{1c}(u)\left[-\alpha I_1(\alpha(t-u))+(\lambda P+\gamma)I_0(\alpha(t-u))\right]du\\ &-\mu\int_0^t P_{11}(u)\exp\{-(\lambda P+\gamma)(t-u)\}I_0(\alpha(t-u))du+\theta\int_0^t\sum_{m=1}^{\infty}P_{0c+m}(u)z^m\\ &\exp\{-(\lambda P+\gamma)(t-u)\}(\beta)^{-m}I_{-m}(\alpha(t-u))du \qquad (15)\end{aligned}$$

As P(z,t) does not contain terms with negative power of z, replacing the right side of equ.13 by -n, we get

$$\begin{aligned}&\exp\{-(\lambda P+\gamma)(t-u)\}I_n(\alpha t)\beta^n+\lambda P\int_0^t\exp\{-(\lambda P+\gamma)(t-u)\}P_{1c}(u)\\ &*\left[\beta^{n-1}I_{n+1}(\alpha(t-u))-I_n(\alpha(t-u))\beta^n\right]du+\int_0^t\exp\{-(\lambda P+\gamma)(t-u)\}q_{1c}(u)\\ &*\left[-\lambda PI_{n+1}(\alpha(t-u))\beta^{n-1}+(\lambda P+\gamma)I_n(\alpha(t-u))\beta^n-\gamma I_{n-1}(\alpha(t-u))\beta^{n+1}\right]du\\ &-\mu\int_0^t P_{11}(u)\exp\{-(\lambda P+\gamma)(t-u)\}\beta^n I_n(\alpha(t-u))du+\theta\int_0^t\sum_{m=1}^{\infty}P_{0c+m}(u)z^m\\ &\exp\{-(\lambda P+\gamma)(t-u)\}(\beta z)^{n-m}I_{n+m}(\alpha(t-u))du \qquad (16)\end{aligned}$$

On simplifying equ.(16) we get,

$$P_{1n+c}(t) = n\beta^n \int_0^t \exp\{-(\lambda P+\gamma)(t-u)\}\left[\frac{I_n(\alpha(t-u))}{(t-u)}\right]P_{1c}(u)$$

$$+\theta\int_0^t \sum_{m=1}^{\infty} P_{0c+m}(u)\exp\{-(\lambda P+\gamma)(t-u)\}(\beta)^{n-m}\left[I_{n-m}(\alpha(t-u))-I_{n+m}(\alpha(t-u))\right]du \qquad (17)$$

Evaluation of $P_{on}(t)$:

By taking Laplace transform of equations (1) (2) and (4) leads to

$$P_{00}^*(s) = \frac{1}{s+\lambda} + \frac{\mu}{s+\lambda}P_{11}^*(s) \qquad ($$

$$P_{0n}^*(s) = \left(\frac{\lambda}{s+\lambda}\right)^n P_{00}^*(s) \qquad 1 \le n < c \qquad (19)$$

$$P_{on}^*(s) = \left(\frac{\lambda P}{s+\lambda P+\theta}\right)^{n+1-c}\left(\frac{\lambda}{s+\lambda}\right)^{c-1} P_{00}^*(s) \qquad n > c \qquad (20)$$

By equations (19) and (20) we get

$$P_{on}(t) = \frac{\lambda^n}{(n-1)!}t^{n-1}e^{-\lambda t}P_{00}(t) \qquad 1 \le n < c \qquad (21)$$

$$P_{on}(t) = \frac{(\lambda P)^{n+1-c}}{(n-c)!}t^{n-c}e^{-(\lambda P+\theta)}\frac{\lambda^{c-1}}{(c-2)!}t^{c-2}e^{-\lambda t}P_{00}(t) \qquad n > c \qquad (22)$$

Sub n=1 in equ(17)

$$P_{11}(t) = \beta\int_0^t \exp\{-(\lambda P+\gamma)(t-u)\}\frac{I_1(\alpha(t-u))}{(t-u)}P_{10}(u)du + \theta\int_0^t\sum_{m=1}^{\infty}P_{om}(y)\beta^{1-m}\exp\{-(\lambda P+\gamma)(t-u)\}$$
$$*\left[I_{m-1}(\alpha(t-u))-I_{m+1}(\alpha(t-u))\right]dy \qquad (23)$$

The corresponding transform is

$$P_{11}^*(s) = \frac{\theta}{\gamma}\left(\frac{\lambda P}{s+\lambda P+\theta}\right)^{1-c}\left(\frac{\lambda}{s+\lambda}\right)^{c-1}\sum_{m=1}^{\infty}\left(\frac{\lambda P/\beta}{s+\lambda P+\theta}\right)^m\left[\frac{p-\sqrt{p^2-\alpha^2}}{\alpha}\right]^m P_{00}^*(s) \qquad (24)$$

$$P_{00}^*(s) = \sum_{j=0}^{\infty}\frac{\theta^j P^j}{c(s+\lambda)^{j+1}(s+\lambda P)^{1-c}}(1+\frac{\theta}{s+\lambda P})^j\left(\sum_{m=1}^{\infty}\left(\frac{\lambda P/\beta}{s+\lambda P+\theta}\right)^m\left[\frac{p-\sqrt{p^2-\alpha^2}}{\alpha}\right]^m\right)^j$$

Which on inversion gives

$$P_{00}(t)=\sum_{j=0}^{\infty}\sum_{r=0}^{\infty}\binom{(N-1)j}{r}\frac{\theta^{r+j}t^{r+j}e^{-\lambda t}e^{-\lambda Pt}}{(r+j)!}$$

$$*\left(\sum_{m=1}^{\infty}\left(\frac{\lambda P}{\beta}\right)^{m}\frac{t^{m-1}e^{-(\lambda P+\theta)t}}{(m-1)!}\frac{mI_m(\alpha t)}{t}e^{-(\lambda P+\mu)t}\right)^{*j}$$

Discussion:

A multi server markovian queuing model with poisson arrival and exponential distribution is analyzed in transient state. The server goes to vacation whenever there is no customer and resumes to service when there are more than c customers and the customer balk the server when there are c or more than c servers. Analytical expression for the state probabilities are obtained by modified Bessel function , the time dependant system size probabilities is found using Laplace transform and the server behavior from time to time is calculated. Based on $P_{11}^{*}(s)$ and $P_{00}^{*}(s)$ various performance measures can be analyzed in future.

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TRANSIENT SOLUTION OF M/M/C QUEUEING MODEL FOR HETEROGENOUS SERVER WITH CATASTROPHES AND BALKING.

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ABSTRACT: *In this paper we analyze M/M/C Queueing model of heterogenous service rate with catastrophes and balking. For this model we consider the arrival follows poisson and the service time follows an exponential distribution. Further the finite C servers may breakdown at any time and the arriving customers may balk with fixed probability. Based on the above considerations, by using probability generating function along with the Bessel properties transient solution of the model is found in a direct way. Moreover, the explicit time dependent probabilities of system wise is also obtained.*

Keywords: *Heterogeneous, Probability generating function, balking, catastrophes, Bessel function.*

INTRODUCTION:

Queueing theory was developed to build models which in turn predicts behavior of systems. Generally, a queueing system can be classified as customer arriving for service, waiting for service and leaving the system after being served. More contributions have been developed in different directions of queueing theory. Parthasarathy [3] explained about the procedure for solving the differential difference equation of a queueing model. Most of the M/M/C queueing models explained in the literature review was assuming that the servers are identical in their service rate. It is also a true in many real-life situations.

Haight [2] introduced the balking concept in M/M/1 queue. On the other hand, Varshney et al [5] discussed about multiserver queueing model with balking and reneging by using of diffusion approximation method. Wang and Chang [6] analyzed the study of M/M/C/N queue with balking, reneging and server breakdown. Indeed, there are several papers dealt with the two phenomena catastrophes and balking with homogenous service rate. Dharmaraja[1] found that the transient solution for markovian queueing model with heterogeneous server and catastrophes. Based on the above literature we have analyzed the transient solution of the multiserver queueing model with heterogenous server of catastrophes and balking effect. By defining the suitable probability generating function we obtain the transient solution of the model.

MODEL DESCRIPTION:

Here we consider a M/M/C queueing model for heterogenous server with the arrival rate which follows a poisson distribution and service rate is μ with exponential distribution. The system may suffer from catastrophes at the rate ε. Whenever a catastrophe occurs in the system, where the C servers become inactivated momentarily and the server become ready for service quickly after the catastrophe. Suppose the customer decides not to join the waiting line is called balking. Here the customer balk with the probability 1-β and join the line of probability β. The system follows a first come first serve {FCFS]. Based on the above criteria, are constructed the following equations of the difference differential equations of our model,

$$\frac{dP_0(t)}{dt} = -(\lambda+\varepsilon)P_0(t) + \mu_1 P_1(t) + \varepsilon \qquad (1)$$

$$\frac{dP_n(t)}{dt} = -\left(\lambda+\varepsilon+\sum_{i=1}^{n}\mu_i\right)P_n(t) + \lambda P_{n-1}(t) + \sum_{i=1}^{n+1}\mu_i P_{n+1}(t), n = 1,2 \dots c-1 \qquad (2)$$

$$\frac{dP_c(t)}{dt} = -(\lambda\beta+\varepsilon+\textstyle\sum_{i=1}^{c}\mu_i)P_c(t) + \lambda P_{c-1}(t) + \textstyle\sum_{i=1}^{c}\mu_i P_{c+1}(t) \qquad (3)$$

$$\frac{dP_n(t)}{dt} = -(\lambda\beta+\varepsilon+\textstyle\sum_{i=1}^{c}\mu_i)P_n(t) + \lambda\beta P_{n-1}(t) + \textstyle\sum_{i=1}^{c}\mu_i P_{n+1}(t), n = c+1 \dots \qquad (4)$$

$$\text{Define } P(z,t) = q_c(t) + \textstyle\sum_{n=1}^{\infty} P_{n+c}(t)z^n \qquad (5)$$

$$P(z,0) = 1 \ \ and \ \ q_c(t) = \textstyle\sum_{n=0}^{c} P_n(t) \qquad (6)$$

Adding eqn (1) – (3) we get

$$\frac{dq_c(t)}{dt} = -\lambda\,\beta P_c(t) + \sum_{i=1}^{c}\mu_i P_{c+1}(t) - \varepsilon q_c(t) + \varepsilon \qquad (7)$$

Now multiplying the equation by z^n summing over the corresponding limit values, we get,

$$\frac{d\sum_{n=1}^{\infty} P_{n+c}(t)z^n}{dt} = [-(\lambda\,\beta + \varepsilon + \sum_{i=1}^{c}\mu_i) + (\lambda\,\beta z + \sum_{i=1}^{c}\frac{\mu_i}{z})]\sum_{n=1}^{\infty} P_{n+c}(t)z^n + \lambda\,\beta z P_c(t) - \sum_{i=1}^{c}\mu_i\, P_{c+1}(t) \qquad (8)$$

Adding eqn (7) & (8) using (5) we get

$$\frac{dP(z,t)}{dt} = [(\lambda\,\beta z + \sum_{i=1}^{c}\frac{\mu_i}{z}) - (\lambda\,\beta + \varepsilon + \sum_{i=1}^{c}\mu_i)]\,P(z,t) + \lambda\,\beta(z-1)P_c(t) - [\left(\lambda\,\beta z + \sum_{i=1}^{c}\frac{\mu_i}{z}\right) - (\lambda\,\beta + \sum_{i=1}^{c}\mu_i)]\,q_c\,(t) + \varepsilon \qquad (9)$$

By Solving the above first order differential equation we get,

$$P(z,t) = e^{-[(\lambda\beta z + \sum_{i=1}^{c}\frac{\mu_i}{z}) - (\lambda\beta + \varepsilon + \sum_{i=1}^{c}\mu_i)]t}\, e^{[(\lambda\beta z + \sum_{i=1}^{c}\frac{\mu_i}{z}) - (\lambda\beta + \varepsilon + \sum_{i=1}^{c}\mu_i)]t} + \int_0^t [\,\lambda\,\beta(z-1)P_c(u) - [\left(\lambda\,\beta z + \sum_{i=1}^{c}\frac{\mu_i}{z}\right) - (\lambda\,\beta + \sum_{i=1}^{c}\mu_i)]\,q_c(u)]e^{[(\lambda\beta z + \sum_{i=1}^{c}\frac{\mu_i}{z}) - (\lambda\beta + \varepsilon + \sum_{i=1}^{c}\mu_i)](t-u)} + \varepsilon e^{-[(\lambda\beta z + \sum_{i=1}^{c}\frac{\mu_i}{z}) - (\lambda\beta + \varepsilon + \sum_{i=1}^{c}\mu_i)](t-u)}\,du \qquad (10)$$

By considering, $\gamma = \sum_{n=1}^{c}\mu_i$, $a = 2\sqrt{\lambda\beta\gamma}$ & $b = \sqrt{\frac{\lambda\beta}{\gamma}}$ then we obtain

$$e^{\left(\lambda\beta z + \frac{\gamma}{z}\right)t} = \sum_{n=-\infty}^{\infty}(bz)^n I_n(at) \qquad (11)$$

By Applying eqn (11) in eqn (10) we get

$$P(z,t) = e^{-(\lambda\beta + \varepsilon + \gamma)t}\sum_{n=-\infty}^{\infty}(bz)^n I_n(at) + \lambda\beta\int_0^t e^{-(\lambda\beta + \varepsilon + \gamma)(t-u)}[\sum_{n=-\infty}^{\infty}(bz)^n[b^{-1}I_{n-1}a(t-u) - I_n a(t-$$

uPcudu+0te−λβ+ε+γt−un=−∞∞(bz)n[−(λβb−1In−1at−u+

(λ+γ)Inat−u−γbIn+1at−u]qcudu+ ε0te−λβ+ε+γt−un=−∞∞(bz)nInat−udu

(12)

Comparing the coefficient of z^n on both the side of eqn (12) we obtain for n=1,2… by using eqn (5)

$$P_{n+c}(t) = e^{-(\lambda\beta + \varepsilon + \gamma)t}b^n I_n(at) + \lambda\beta\int_0^t e^{-(\lambda\beta + \varepsilon + \gamma)(t-u)}\,[b^{n-1}I_{n-1}a(t-u) - b^n I_n a(t-u)]P_c(u)du - \int_0^t e^{-(\lambda\beta + \varepsilon + \gamma)(t-u)}\,[\lambda\beta b^{n-1}I_{n-1}a(t-u) - (\lambda+\gamma)b^n I_n a(t-u) + \gamma b^{n+1}I_{n+1}a(t-u)]q_c(u)du + \varepsilon\int_0^t e^{-(\lambda\beta + \varepsilon + \gamma)(t-u)}b^n I_n a(t-u)du \qquad (13)$$

Comparing the constant terms in eqn (12) and substituting n=0 we get

$$q_c(t) = e^{-(\lambda\beta + \varepsilon + \gamma)t}I_0(at) + \lambda\beta\int_0^t e^{-(\lambda\beta + \varepsilon + \gamma)(t-u)}\,[b^{-1}I_1 a(t-u) - I_0 a(t-u)]P_c(u)du + \int_0^t e^{-(\lambda\beta + \varepsilon + \gamma)(t-u)}\,[-aI_1 a(t-u) + (\lambda+\gamma)I_0 a(t-u)]q_c(u)du + \varepsilon\int_0^t e^{-(\lambda\beta + \varepsilon + \gamma)(t-u)}I_0 a(t-u)du \qquad (14)$$

Eqn (13) implies that

$$\int_0^t e^{-(\lambda\beta+\varepsilon+\gamma)(t-u)}\,[-\lambda\beta I_{n+1}a(t-u)b^{n-1}+(\lambda+\gamma)b^n I_n a(t-u)-\gamma b^{n+1}I_{n-1}a(t-u)]q_c(u)du$$
$$= e^{-(\lambda\beta+\varepsilon+\gamma)t}b^n I_n(at)$$
$$+\lambda\beta\int_0^t e^{-(\lambda\beta+\varepsilon+\gamma)(t-u)}\,[b^{n-1}I_{n+1}a(t-u)-b^n I_n a(t-u)]P_c(u)du$$
$$+\varepsilon\int_0^t e^{-(\lambda\beta+\varepsilon+\gamma)(t-u)}b^n I_n a(t-u)du \tag{15}$$

Using (15) in eqn (13) we get $P_n(t)$ implies that

$$P_{n+c}(t) = nb^n\int_0^t e^{-(\lambda\beta+\varepsilon+\gamma)(t-u)}\left[\frac{I_n a(t-u)}{(t-u)}\right]P_c(u)du \tag{16}$$

The remaining probabilities $P_n(t)$ $n = 0,1,2 \dots c$ can be found by solving the system of equations (1)& (2).

The matrix form of eqn (1) & (2) is $\frac{dP(t)}{dt} = BP(t) + \gamma P_c(t)e_1 + \varepsilon e_2$ (17)

Where the matrix $B = (b_{ij})_{(c\times c)}$ is given as

$$\begin{bmatrix} -(\lambda\beta+\varepsilon) & \mu_1 & \dots & 0 \\ \lambda\beta & -(\lambda\beta+\varepsilon+\mu_1) & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & & \sum_{i=1}^{c-1}\mu_i \\ 0 & 0 & & -(\lambda\beta+\varepsilon+\sum_{i=1}^{c-1}\mu_i) \end{bmatrix}$$

$P(t) = \left(P_0(t), P_1(t) \dots P_{c-1}(t)\right)^T$ $e_1 = (0,0 \dots 1)^T$ $and\ e_2 = (1,0 \dots 0)^T$ are column vectors of order C. $P^*(s)$ denote the laplace transforms of $P(t)$.

By taking the laplace transforms of eqn (17) and solving of $P^*(s)$, we get

$$P^*(s) = \left[\gamma P_c{}^*(s)e_1 + P(0) + \frac{\varepsilon}{s}e_2\right][SI-B]^{-1} \tag{18}$$

Thus $e^T P^*(s) + P_c{}^*(s) = q_c{}^*(s)$ (19)

Define $f(s) = [(s+\lambda\beta+\gamma+\varepsilon) - \sqrt{(s+\lambda\beta+\gamma+\varepsilon)^2 - a^2}]$

Taking laplace of eqn (14) and solving for $q_c{}^*(s)$ we get,

$$s(s+\varepsilon)q_c{}^*(s) = (s+\varepsilon) + sP_c{}^*(s)\frac{1}{2}[f(s)-ab] \tag{20}$$

Using eqn (20) in eqn (19) we get,

$$P_c{}^*(s) = \frac{(s+\varepsilon)}{s}\left(\frac{1-se^T[SI-B]^{-1}\left(P(0)+\frac{\varepsilon}{s}e_2\right)}{(s+\varepsilon+\lambda\beta)-\frac{1}{2}f(s)+\gamma e^T[SI-B]^{-1}e_1(s+\varepsilon)}\right) \tag{21}$$

Raju and Bhat [4] gives the procedure for finding the elements of the matrix$[SI-B]^{-1}$.

Let $[SI-B]^{-1} = (d_{ij}{}^*(s))_{c\times c}$

Here $[SI-B]^{-1}$is almost lower triangular matrix.

$$d_{ij}{}^*(s) = \begin{cases} \dfrac{1}{\sum_{k=1}^{j+1}\mu_k}\ \dfrac{u_{c,j+1}(s)u_{i,0}(s)-u_{i,j+1}(s)u_{c,0}(s)}{u_{c,0}(s)} & ,j = 0,1,\dots c-2 \\ \dfrac{u_{i,0}(s)}{u_{c,0}(s)} & j = c-1. \end{cases} \tag{22}$$

$u_{i,j}(s)$ are given as,

$$u_{i,j} = 1 \qquad i = 0,1 \dots c-1.$$
$$u_{i+1,i} = \frac{s+\lambda\beta+\varepsilon+\sum_{k=1}^{i}\mu_k}{\sum_{k=1}^{i+1}\mu_k} \qquad i = 0,1 \dots c-2.$$
$$u_{i+1,i-j} = \frac{(s+\lambda\beta+\varepsilon+\sum_{k=1}^{i}\mu_k}{\sum_{k=1}^{i+1}\mu_k}, \qquad j \le i, i = 1,2 \dots c-2.$$

$$u_{c,j} = \begin{cases} (s+\lambda\beta+\varepsilon+\sum_{k=1}^{c-1}\mu_k)u_{c-1,j} - \lambda u_{c-2,j} \ , & j = 0,1 \dots c-2. \\ (s+\lambda\beta+\varepsilon+s+\lambda\beta+\varepsilon+\sum_{k=1}^{c-1}\mu_k) \ , & j = c-1. \end{cases} \tag{23}$$

For other i&j $u_{i,j} = 0$

Eqn (21) implies that $P_c{}^*(s) = \frac{(s+\varepsilon)}{s}\left(\frac{(1-(s+\varepsilon)\sum_{i=0}^{c-1} b_{i0}{}^*(s)}{[(s+\varepsilon+\lambda\beta)-\frac{1}{2}f(s)+\gamma(s+\varepsilon)\sum_{j=0}^{c-1} b_{j,c-1}{}^*(s)]}\right)$ (24)

Eqn (18) implies that

$P_k{}^*(s) = \left(1+\frac{\varepsilon}{s}\right) b_{k,0}{}^*(s) + \gamma b_{k,c-1}{}^*(s) P_c{}^*(s)$ (25)

Since $e_1 = (0,0 \dots 1)^T$ $\quad$ *and* $e_2 = (1,0 \dots 0)^T$

$$P_c{}^*(s) = \frac{(1+\frac{\varepsilon}{s})E^*(s)}{\frac{1}{2}f(s)\left[1-\frac{2\gamma(1-F^*(s))}{(s+\lambda\beta+\gamma+\varepsilon)-\sqrt{(s+\lambda\beta+\gamma+\varepsilon)^2-a^2}}\right]} \tag{26}$$

$$E^*(s) = \sum_{i=0}^{c-1}\frac{E_i}{s-s_i} \tag{27}$$

$$\text{and } F^*(s) = \sum_{i=0}^{c-1}\frac{F_i}{s-s_i} \tag{28}$$

Here $E_i = \lim_{s\to s_i} s - s_i[1-\sum_{l=0}^{c-1}(s+\varepsilon)b_{lo}{}^*(s)$ (29)

$$F_i = \lim_{s\to s_i} s - s_i[\sum_{l=0}^{c-1}(s+\varepsilon)b_{l,c-1}{}^*(s)\,] \tag{30}$$

Eqn (26) implies that

$$P_c{}^*(s) = \sum_{n=0}^{\infty}\sum_{m=0}^{n}(n+1)\frac{(-1)^m}{\gamma}\left(\frac{a}{2\lambda\beta}\right)^{n+1}\binom{n}{k}$$
$$\left[\left(1+\frac{\varepsilon}{s}\right)E^*(s)(F^*(s))^m\frac{[(s+\lambda\beta+\gamma+\varepsilon)-\sqrt{(s+\lambda\beta+\gamma+\varepsilon)^2-a^2}]^{n+1}}{(n+1)a^{n+1}}\right] \tag{31}$$

Taking laplace inverse of eqn(31) we obtain

$$P_c(t) = \sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{(-1)^m}{\gamma}\left(\frac{a}{2\lambda\beta}\right)^{n+1}\binom{n}{k}$$
$$\left[\int_0^t E(t-u)\int_0^u F^{c(m)}(u-v)e^{-(\lambda\beta+\gamma+\varepsilon)v}\frac{I_{n+1}(av)}{v}dudv\right]$$
$$+\varepsilon\left[\int_0^t G(t-u)\int_0^u F^{c(m)}(u-v)e^{-(\lambda\beta+\gamma+\varepsilon)v}\frac{I_{n+1}(av)}{v}dudv\right] \tag{32}$$

Here $G(t) = \int_0^t E(u)du$ *and* $F^{c(m)}$is m-fold convolution of $F(t)$ with $F^{c(0)} = \delta(t)$ is a direct delta function.

Laplace inverse of eqn (25) implies that

$P_k(t) = b_{k0}(t) + \varepsilon\int_0^t b_{k0}(u)\,du + \gamma\int_0^t b_{kc-1}(u)\,P_c(t-u)du\,, \quad k = 0,1 \dots c-1$ (33)

Thus eqn (16), (32) & (33) are the state probability values of our model. Hence the time-dependent values are obtained.

CONCLUSION:

In this paper we have established the time dependent solution of the heterogenous M/M/C queueing model with catastrophes and balking using probability generating function technique. In future, with the aid of transient state probability, the various performance measures can be analyzed.

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On b-chromatic number of Net graph

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ABSTRACT: *In this paper, we obtain the b-chromatic number for the net graph N, Line graph of net graph $L(N)$,central graph of net graph $C(N)$, middle graph of net graph $M(N)$, total graph of net graph $T(N)$.*

Keywords: *Line graph, Middle graph, Central graph, Total graph, Net graph, b-chromatic number.*

1.INTRODUCTION

All graphs considered in this paper are nontrivial, simple and undirected. Let G be a graph with vertex set V and edge set E. A *k*-coloring of a graph G is a partition $P=\{V_1,V_2,V_3,...,V_k\}$ of V into independent sets of G. The minimum cardinality k for which G has a *k*-coloring is the chromatic number $\chi(G)$ of G. A *b*-coloring of a graph G is a proper coloring of the vertices of G such that there exists a vertex in each color class joined to at least one vertex in each other color class. The *b*-chromatic number of a graph G, denoted by $\varphi(G)$ is the maximal integer k such that G may have a b-coloring with k colors. The b-chromatic number was introduced by Irving and Manlove [2] by considering proper colourings that are minimal with respect to a partial order defined on the set of all partitions of $V(G)$. There has been an increasing interest in the study of *b*-coloring since the publication of [2]. Irving and Manlove [2] have also proved the following upper bound of $\varphi(G)$

$$\varphi(G)\le\Delta(G)+1$$

Jakovac and Klavzar [2], proved that the *b*-chromatic number of cubic graphs is four with the exception of Petersen graph, $K_{3,3}$, prism over K_3, and sporadic with 10 vertices.

2. PRELIMINARIES

Definition (1) The net graph or 3-sunlet graph is the graph on 6 vertices obtained by attaching 3 pendent edges to a cycle graph C_3 and is denoted by N.

Definition (2) The line graph [16] of a graph G, denoted by $L(G)$, is a graph whose vertices are the edges of G, if $u,v\in E(G)$ then $uv\in E(L(G))$ if u and v share a vertex in G.

Definition (3) A graph is said to be a central graph of $G=(V,E)$ an operation is done on G, by dividing each edge of G once and joining all the non-adjacent vertices of G and is denoted by $C(G)$

Definition (4) Consider the vertex set $V(G)$ and the edge set $E(G)$ in a graph G. The middle graph of G, denoted by $M(G)$ is defined in the following way. $V(G)\cup E(G)$ be the vertex set of $M(G)$. consider x,y be two vertices of $M(G)$ are adjacent in $M(G)$ in case one of the following conditions holds:

(i) x,y are in $E(G)$ and x,y are adjacent in G.

(ii) x is in $V(G)$, y is in $E(G)$,and x,y are incident in G.

Definition (5) Consider the vertex set $V(G)$ and the edge set $E(G)$ in a graph G. The total graph of G, denoted by $T(G)$ is defined as follows. $V(G) \cup E(G)$ be the vertex set of $M(G)$. Consider x, y be two vertices of $M(G)$ are adjacent in $M(G)$ in case one of the following conditions holds:

(i) x, y are in $V(G)$ and x is adjacent to y in G.

(ii) x, y are in $E(G)$ and x, y are adjacent in G.

(iii) x is in $V(G)$, y is in $E(G)$,and x, y are incident in G.

Theorem 1. The *b*-chromatic number of net graph is $\varphi(N) = 3$.

Proof.

Let N be the net graph on 6 vertices. Consider $V(S_n) = \{v_1, v_2, ..., v_n\} \cup \{u_1, u_2, ..., u_n\}$ every $v_i u_i$ is a pendant edge when v_i's are the vertices of cycles taken in cyclic order and u_i's are pendant vertices. Consider the 3-coloring (c_1, c_2, c_3) of S_n, assign the color c_2 to v_1, c_3 to v_2 ,c_1 to v_3 if any, assign the allowed colors, because $\deg(v_i) = 3$, which shows that the coloring is a *b*-coloring. Therefore, $\varphi(N) = 3$.

Theorem 2. The *b*-chromatic number of the line graph of net graph is $\varphi(L(N)) = 4$.

Proof.

Let N be the net graph on 6 vertices. Consider $V(S_n) = \{v_1, v_2, ..., v_n\} \cup \{u_1, u_2, ..., u_n\}$ and $E(S_n) = \{e_i' : 1 \le i \le n\} \cup \{e_i : 1 \le i \le n-1\} \cup \{e_n\}$ where e_i' is the edge of $v_i u_i (1 \le i \le n)$, e_i is the edge $v_i v_{i+1} (1 \le i \le n-1)$ and e_n is the edge $v_n v_1$. By the line graph definition $V(L(N)) = E(S_n) = \{u_i' : 1 \le i \le n\} \cup \{v_i' : 1 \le i \le n-1\} \cup \{V_n'\}$ where v_i' represent the edge e_i and u_i' represent the edge $e_i' (1 \le i \le n)$. Assume the following 4-coloring (c_1, c_2, c_3, c_4) of $L(N)$, assign the color c_2 to u_1', c_3 to u_2', c_2 to u_3', c_1 to v_1', c_2 to v_2' and c_4 to v_3' if any, assign the allowed colors, because $\deg(v_i') = 4$, which shows that the coloring is a *b*-coloring. Therefore, $\varphi(L(N)) = 4$.

Theorem 3. The *b*-chromatic number of the middle graph of net graph is $\varphi(M(N)) = 4$.

Proof.

Let N be the net graph on 6 vertices. Consider $V(S_n) = \{v_1, v_2, ..., v_n\} \cup \{u_1, u_2, ..., u_n\}$ and $E(S_n) = \{e_i' : 1 \le i \le n\} \cup \{e_i : 1 \le i \le n-1\} \cup \{e_n\}$ where e_i' is the edge of $v_i u_i (1 \le i \le n)$, e_i is the edge $v_i v_{i+1} (1 \le i \le n-1)$ and e_n is the edge $v_n v_1$. By the middle graph definition $V(M(N)) = V(S_n) \cup E(S_n) = \{v_i : 1 \le i \le n\} \cup \{u_i : 1 \le i \le n\} \cup \{v_i' : 1 \le i \le n\} \cup \{u_i' : 1 \le i \le n\}$ where v_i' represent the edge e_i and u_i' represent the edge $e_i' (1 \le i \le n)$. Assume the following 4-coloring (c_1, c_2, c_3, c_4) of $L(N)$, assign the color c_2 to u_1', c_3 to u_2', c_2 to u_3', c_1 to v_1', c_2 to v_2' and c_4 to v_3' if any, assign the allowed colors, because $\deg(u_i') = 1$,if any vertex v_i' then assign the allowed colors, because $\deg(v_i') = 6$ which shows that the coloring is a *b*-coloring. Therefore, $\varphi(M(N)) = 4$.

Theorem 4. The *b*-chromatic number of the central graph of net graph is $\varphi(C(N)) = 3$.

Proof.

Let N be the net graph on 6 vertices. By the central graph definition we have to consider $V(S_n) = \{v_1, v_2, ..., v_n\} \cup \{u_1, u_2, ..., u_n\}$ v_i's and u_i's be the vertices of the complete graph. The vertices of the complete graph is in the cyclic order. In $C(N)$, consider the vertex on the edge joining v_i and v_{i+1} be $v_{i,i+1}$ for $i = 1,2,3,..., n-1$, the vertex on the edge joining v_1 and v_n be $v_{1,n}$, the vertex on the edge joining u_i and v_i be $u_i v_i$ for $i = 1,2,3,..., n$. Assume the following 3-coloring (c_1, c_2, c_3) of $L(N)$, assign

the color $c_2 \, to \, u_1$, $c_3 \, to \, u_2$, $c_2 \, to \, u_3$, $c_1 \, to \, v_1$, $c_2 \, to \, v_2$ and $c_3 \, to \, v_3$ if any, assign the allowed colors, because $\deg(u_i^{'}) = 1$,if any vertex $v_i^{'}$ then assign the allowed colors, because $\deg(v_i) = 4$ which shows that the coloring is a *b*-coloring. Therefore, $\varphi(C(N)) = 3$.

Theorem 5. The *b*-chromatic number of the total graph of net graph is $\varphi(T(N)) = 4$.

Proof.

Let N be the net graph on 6 vertices. Assume the following 4-coloring (c_1, c_2, c_3, c_4) of $T(N)$, assign the color $c_2 \, to \, u_1^{'}$, $c_3 \, to \, u_2^{'}$, $c_2 \, to \, u_3^{'}$, $c_1 \, to \, v_1^{'}$, $c_2 \, to \, v_2^{'}$ and $c_4 \, to \, v_3^{'}$, $c_2 \, to \, u_1$, $c_3 \, to \, u_2$, $c_2 \, to \, u_3$, $c_1 \, to \, v_1$, $c_2 \, to \, v_2$ and $c_3 \, to \, v_3$ if any, assign the allowed colors, because $\deg(v_i^{'}) = 6$ & $\deg(u_i^{'}) = 1$,if any vertex $v_i^{'}$ then assign the allowed colors, because $\deg(v_i) = 4$ which shows that the coloring is a *b*-coloring. Therefore, $\varphi(T(N)) = 4$.

In this research paper, we find out the b-chromatic number of line graph, middle graph and total graph of net graph are equal to 4 and the b-chromatic number of central graph are equal to 3.

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ANTI FUZZY MEET SEMI L-FILTER

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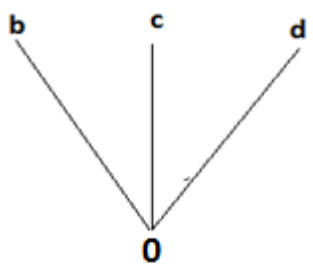
ABSTRACT: *In this paper we have made an attempt to define anti fuzzy meet semi L-Filter and anti fuzzy level meet semi L- Filter. We have stated the properties of anti fuzzy meet semi L- Filter, anti fuzzy level meet semi L- Filter. We introduce some theorems, propositions and Lemmas in these concepts and we have given proofs for the same.*

Keywords: *Meet semi lattices, anti fuzzy meet semi L- Filter, anti fuzzy level meet semi L-Filter.*

Introduction: The concept of a fuzzy set was published in 1965 ,by Lotfi A. Zadeh[6]. Further more, the idea of anti fuzzy subgroups was introduced by Biswas[3] which ultimately paved way to many researchers.In this paper we define anti-fuzzy meet semi L-filter and anti-fuzzy level meet semi L-filter.The theorems and lemmas under these concepts were discussed.

Definition 1: Let A be a meet semilattice. An Anti fuzzy meet subsemilattice $T_\mu : A \to [0,1]$ is called an anti Fuzzy meet semi L-Filter of A if $\forall x, y \in A,\ T_\mu(x \wedge y) \geq \max\{T_\mu(x), T_\mu(y)\}$

Example 1: Consider A={0,a,b,c}.Let $T_\mu : A \to [0,1]$ an be a meet semilattice in A defined by $T_\mu(0) = 0.9, T_\mu(a) = 0.7, T_\mu(b) = 0.5, T_\mu(c) = 0.3$



Thus T_μ is an anti fuzzy meet semi L- filter.

Definition 2: Let $T_{\mu 1}$ and $T_{\mu 2}$ be any two anti meet semi L- Filter of a meet semi lattice A. Then $T_{\mu 1}$ is said to be contained in $T_{\mu 2}$ if $T_{\mu 1}(x) \leq T_{\mu 2}(x), \forall x \in A$ and is denoted by $T_{\mu 1} \subseteq T_{\mu 2}$.

Definition 3: Let $T_{\mu 1}$ and $T_{\mu 2}$ be any two anti fuzzy meet semi L- Filter of a meet semi lattice A. If $T_{\mu 1}(x) = T_{\mu 2}(x), \forall x \in A$, then $T_{\mu 1}$ and $T_{\mu 2}$ are said to be equal and it is written as $T_{\mu 1} = T_{\mu 2}$

Definition 4: The complement of a anti fuzzy meet semi L- Filter T_μ of a meet semilattice A is symbolized by $\sim T_\mu(x) = 1 - T_\mu(x), \forall x \in A$.

Properties: Let A be a meet semilattice and B be a meet subset of A. Define $\chi_B : A \to [0,1]$ as

$$\chi_B(x) = \begin{cases} 1 & if \quad x \in B \\ 0 & if \quad x \notin B \end{cases}$$

Then the following properties hold:

1. χ_ϕ and χ_A denote constant functions from A to 0 and 1 respeactively.

2. $T_\mu \cap \chi_\phi = \chi_\phi$.
3. $T_\mu \cup \chi_\phi = T_\mu$.
4. $T_\mu \cap \chi_A = T_\mu$.
5. $T_\mu \cup \chi_A = \chi_A$.

Definition 5: The intersection of two anti fuzzy meet semi L- Filter $T_{\mu 1}$ and $T_{\mu 2}$ of a meet semi lattice A is defined as $[T_{\mu 1} \cap T_{\mu 2}](x) = \min\{T_{\mu 1}(x), T_{\mu 2}(x)\}, \forall x \in A$

Definition 6: The union of two anti fuzzy meet semi L- Filter $T_{\mu 1}$ and $T_{\mu 2}$ of a meet semilattice A is defined as $[T_{\mu 1} \cup T_{\mu 2}](x) = \max\{T_{\mu 1}(x), T_{\mu 2}(x)\}, \forall x \in A$

FUZZY LEVEL MEET SEMI L-FILTER

Definition 7:

Let T_μ be any anti fuzzy meet semi L- Filter of a meet semi lattice A and Let $t \in [0,1]$. Then $T_{\mu t} = \{x \in A / T_\mu(x) \geq t\}$ is called anti fuzzy level meet semi L- Filter of T_μ.

Definition 8:

Let T_μ be a anti fuzzy meet semi L- Filter of a meet semi lattice A. Then the fuzzy level meet semi L-Filter are defined by

$T_{\mu t} = \{x \in A / T_\mu(x) \geq t\}$

$T_{\mu s} = \{x \in A / T_\mu(x) \geq s\}$

Clearly, $T_{\mu t} \subseteq T_{\mu s}$, ,whenever t > s.

Lemma 1: Let T_μ be an anti fuzzy meet semil L- Filter of a meet semilattice A and let $t,s \in \operatorname{Im} T_\mu$.Then $T_{\mu t} = T_{\mu s}$ if t=s.

Proof:

If t=s ,then $T_{\mu t} = T_{\mu s}$.

Conversely,

Let $T_{\mu t} = T_{\mu s}$.

Since $t \in \operatorname{Im} T_\mu$, $\exists$ $x \in A$ such that $T_\mu(x) = t$,

$\Rightarrow t \in T_{\mu s}$

Hence t= $T_\mu(x) \geq s$ ---------- (1)

Similarly,

it can be proved that $s \geq t$---------- (2)

Then from (1) and (2), t=s.

Proposition 1: Let T_μ ,T_σ and T_θ be anti fuzzy meet semi L- Filter of a meet semilattice A.Then the following are true.

(i) Commutativity:

$$T_\mu \cup T_\sigma = T_\sigma \cup T_\mu$$

$$T_\mu \cap T_\sigma = T_\sigma \cap T_\mu$$

(ii) Associativity:

$$T_\mu \cup [T_\sigma \cup T_\theta] = [T_\mu \cup T_\sigma] \cup T_\theta$$

$$T_\mu \cap [T_\sigma \cap T_\theta] = [T_\mu \cap T_\sigma] \cap T_\theta$$

(iii) Idempotency:

$$T_\mu \cup T_\mu = T_\mu$$
$$T_\mu \cap T_\mu = T_\mu$$

(iv) Distributivity:

$$T_\mu \cup [T_\sigma \cap T_\theta] = [T_\mu \cup T_\sigma] \cap [T_\mu \cup T_\theta]$$
$$T_\mu \cap [T_\sigma \cup T_\theta] = [T_\mu \cap T_\sigma] \cup [T_\mu \cap T_\theta]$$

(v) Identity:

$$T_\mu \cup \chi_\phi = T_\mu \text{ and } T_\mu \cap \chi_\phi = \chi_\phi$$
$$T_\mu \cup \chi_A = \chi_A \text{ and } T_\mu \cap \chi_A = T_\mu$$

(vi) Absorption:

$$T_\mu \cup [T_\mu \cap T_\sigma] = T_\mu$$
$$T_\mu \cap [T_\mu \cup T_\sigma] = T_\mu$$

(vii) Demorgan's Law:

(1) $\sim [T_\mu \cap T_\sigma] = [\sim T_\mu] \cup [\sim T_\sigma]$

(2) $\sim [T_\mu \cup T_\sigma] = [\sim T_\mu] \cap [\sim T_\sigma]$

(viii) Involution:

$$\sim (\sim T_\mu) = T_\mu$$

Remark: Let C be any fuzzy subset of a meet semilattice A and T_μ be any anti fuzzy meet semi L- Filter of A. The the following properties hold.

(1) $C \cap (\sim C) = \phi$ and $C \cup (\sim C) = A$

(2) $T_\mu \cap (\sim T_\mu) \neq \chi_\phi$ and $T_\mu \cup (\sim T_\mu) \neq \chi_A$

Theorem 1: Two anti fuzzy meet semi L- Filter T_μ and T_θ of a meet semilattice A such that the card Im $T_\mu < \infty$ and card Im $T_\theta < \infty$ are equal iff Im T_μ =Im T_θ and $F_{S\mu} = F_{S\theta}$.

Proof: Let T_μ and T_θ be two anti fuzzy meet semi L- Filter of a meet semilattice A such that the card Im $T_\mu < \infty$ and card Im $T_\theta < \infty$.

Assume that T_μ and T_θ are equal

(ie) $T_\mu(x) = T_\theta(x)$ ---------- (1)

$\Rightarrow T_\mu(x) \in \operatorname{Im} T_\mu$

$\Rightarrow T_\theta(x) \in \operatorname{Im} T_\mu$

$But T_\theta(x) \in \operatorname{Im} T_\theta$

$\Rightarrow \operatorname{Im} T_\mu \subseteq \operatorname{Im} T_\theta$ ----------(2)

Similarly ,

it can be proved that $\operatorname{Im} T_\theta \subseteq \operatorname{Im} T_\mu$ ------- (3)

(2) and (3) $\Rightarrow \operatorname{Im} T_\mu = \operatorname{Im} T_\theta$ -------(4)

$Let T_{\mu t} \in F_{S\mu} and, x \in T_{\mu t}$

$\Rightarrow T_\mu(x) \geq t, t \in \operatorname{Im} T_\mu$

$\Rightarrow T_\theta(x) \geq t, t \in \operatorname{Im} T_\theta, by(1) and (4).$

$\Rightarrow x \in T_{\theta t}$

$\Rightarrow T_{\mu t} \subseteq T_{\theta t}$

Similarly, it can be proved that $T_{\theta t} \subseteq T_{\mu t}$

$$Hence, T_{\mu t} = T_{\theta t}$$

$$\Rightarrow T_{\theta t} \in F_{S\mu}$$

$$But, T_{\theta t} \in F_{S\theta}$$

$$\Rightarrow F_{S\theta} \subseteq F_{S\mu} ----------(5)$$

$$Similarly, F_{S\mu} \subseteq F_{S\theta} ----------(6)$$

$$(5)\,\&\,(6) \Rightarrow F_{S\mu} = F_{S\theta} ----------(7)$$

Equation (4) and (7) completes the proof.

Conversely, assume that Im T_μ =Im T_θ and $F_{S\mu} = F_{S\theta}$

To prove: T_μ and T_θ are equal.

Suppose $T_\mu(x) \neq T_\theta(x)$ for $x \in A$

Then either Im $T_\mu \neq$ Im T_θ or $F_{S\mu} \neq F_{S\theta}$.

This is a contradiction

Hence $T_\mu(x) = T_\theta(x), \forall x \in A.$

Therefore $T_\mu and, T_\theta$ are equal.

Theorem 2:

The intersection of two anti fuzzy meet semi L Filter Filter of a meet semilattice A is also an anti fuzzy meet semi L ideal Filter of A.

Proof:

Let A be a meet semilattice.

Let $T_{\mu 1}$ and $T_{\mu 2}$ be any two anti fuzzy meet semi L Filter of A.

T.P: $T_{\mu 1} \cap T_{\mu 2}$ is an anti fuzzy meet semi L Filter of A.

Let a,b $\in T_{\mu 1} \cap T_{\mu 2}$

Then a,b $\in T_{\mu 1}$ and a,b $\in T_{\mu 2}$.

$$\Rightarrow a \wedge b \in T_{\mu 1} and, a \wedge b \in T_{\mu 2}$$

Hence $a \wedge b \in T_{\mu 1} \cap T_{\mu 2}$.

Therefore $T_{\mu 1} \cap T_{\mu 2}$ is an anti fuzzy meet semi L Filter of A.

Proposition 2:

Let A be any meet semi lattice. Let I ={T_μ / T_μ is a anti fuzzy meet semi L- Filter of A}. Then for any x $\in$ A, T_μ*(x) = sup {k /x $\in T_{\mu k}$;$T_\mu \in$ I} is a anti fuzzy meet semi L- Filter.

Proof

Let T_α = sup {k /x $\in T_{\mu k}$, $T_\mu \in$ I } and let $\in$ > 0 be arbitrary.

Then $T_\alpha - \in <$ sup {k / x $\in T_{\mu k}$, $T_\mu \in$ I} = k_1 (say)

$\Rightarrow T_\alpha - \in < k_1$, for some k such that x $\in T_{\mu k1}$

$\Rightarrow T_\alpha - \in < T_\mu(x)$, since $T_\mu(x) \geq k_1$,

$\Rightarrow T_\alpha \geq T_\mu(x)$----------(1) since $\in$ > is arbitrary.

If T_μ (x) = t, then x $\in T_{\mu t}$,and also t = sup{k / x$\in T_{\mu k}$}

$t - \in \leq$ sup {k / x$\in T_{\mu k}$}

Hence t $\leq$ sup {k /x$\in T_{\mu k}$}, since $\in$ is arbitrary.

$\Rightarrow t \leq T_\alpha$ ..... (2)

Therefore (1) and (2) $\Rightarrow T_\mu(x) = T_\alpha$

Hence $T_\mu^*(x) = T_\mu(x)$, for some $T_\mu \in I$.
Therefore $T_\mu^*(x \wedge y) \geq \max\{T_\mu^*(x), T_\mu^*(y)\}$ for some $T_\mu \in I$.

Theorem : 3

If T_μ and T_α are any two anti fuzzy meet semi L- Filter of a meet semi lattice A, then $[T_\mu \wedge T_\alpha](x \wedge y) \geq \max\{T_\mu(x), T_\alpha(y)\}$

Proof:

Let $T_\theta = T_\mu \wedge T_\alpha$

$T_\theta(x \wedge y) \geq \max\{T_\theta(x), T_\theta(y)\}$

$= \max\{[T_\mu \wedge T_\alpha](x), [T_\mu \wedge T_\alpha](y)\}$

$=\max\{\min\{\max\{T_\mu(a),T_\alpha(b)\}\},$
$x=a\wedge b$
$\min\{\max\{T_\mu(c),T_\alpha(d)\}\}\}$
$x=c\wedge b$

$= \max\{\{\max\{T_\mu(a), T_\alpha(b)\}, \{\max\{T_\mu(c), T_\alpha(d)\}\}$

$= \max\{T_\mu(x), T_\alpha(y)\}$, where $x=a\wedge b$, $y = c\wedge b$

Therefore, $[T_\mu \wedge T_\sigma](x \wedge y) \geq \max\{T_\mu(x), T_\alpha(y)\}$

Theorem: 4

The intersection of two anti fuzzy level meet semi L- Filter of a meet semi lattice A is also a anti fuzzy level meet semi L- Filter of A.

Proof :

Let T_μ be a anti fuzzy meet semi L- Filter of a meet semilattice A.
Let $T_{\mu t}$, and $T_{\mu s}$ be two anti fuzzy level meet semi L- Filter of a meet semilattice A.
Let $x, y \in T_{\mu t} \cap T_{\mu s}$
Then $x, y \in T_{\mu t}$ and $x \in T_{\mu s}$
$\Rightarrow x \wedge y \in T_{\mu t}$
Also $x \wedge y \in T_{\mu s}$
$\Rightarrow x \wedge y \in T_{\mu t} \cap T_{\mu s}$
There $T_{\mu t} \cap T_{\mu s}$ is a anti fuzzy level meet semi L- Filter of A.

Remark :

The intersection of any family of anti fuzzy level meet semi L- Filter of A is also a anti fuzzy level meet semi L- Filter of A.

Theorem :5

The union of two anti fuzzy level meet semi L- Filter of A is also a anti fuzzy level meet semi L- Filter of A.

Proof:

Let T_μ be a anti fuzzy meet semi L- Filter of A.
Let $T_{\mu t}$ and $T_{\mu s}$ be two anti fuzzy level meet semi L- Filter of A.
Let $x, y \in T_{\mu t} \cup T_{\mu s}$
Then $x, y \in T_{\mu t}$ or $x, y \in T_{\mu s}$
$\Rightarrow x \wedge y \in T_{\mu t} \cup T_{\mu s}$
There $T_{\mu t} \cup T_{\mu s}$ is a anti fuzzy level meet semi L- Filter of A.

Remark :

The union of any family of fuzzy level meet semi L- Filter of A is also a fuzzy level meet semi L- Filter of A.

Theorem : 6

Let A be a meet semilattice. Two anti fuzzy level meet semi L- Filter $T_{\mu s}$ and $T_{\mu t}$ (with $t < s$) of a anti fuzzy meet semi L- Filter T_μ of A are equal iff there if no $x \in A$ such that $s \leq T_\mu(x) < t$.

Proof:

Let $T_{\mu s}$ and $T_{\mu t}$ be two fuzzy level meet Semi L- Filter of a meet semilattice A, where $s<t$.
Assume that $T_{\mu s}$ and $T_{\mu t}$ are equal.
To prove : There is no x in A such that $s \leq T_\mu(x) < t$.
On the contrary, assume that $s \leq T_\mu(x) < t$.for some $x \in A$

$\Rightarrow T_\mu(x) \geq s$ and $T_\mu(x) < t$
$\Rightarrow x \in T_{\mu s}$ and $x \notin T_{\mu t}$
$\Rightarrow T_{\mu s} \neq T_{\mu t}$
This is a contradiction to the assumption.
Hence there is no x in A such that $s \leq T_\mu(x) < t$.
Conversely, assume that there is no x in A such that $s \leq T_\mu(x) < t$ …….. (1)
$T_{\mu s} = \{x \in A / T_\mu(x) \geq s\}$ and
$T_{\mu t} = \{x \in A / T_\mu(x) \geq t\}$ and $t > s$
Then $T_{\mu t} \subseteq T_{\mu s}$……(2)
It is enough to show that $T_{\mu s} \subseteq T_{\mu t}$
Let $x \in T_{\mu s}$
Then $T_\mu(x) \geq s$
$\Rightarrow T_\mu(x) \geq t$ by (1)
$\Rightarrow x \in T_{\mu t}$
$\Rightarrow T_{\mu s} \subseteq T_{\mu t}$ …… (3)
Therefore (2) and (3) imply $T_{\mu s} = T_{\mu t}$
Hence the two anti fuzzy level meet semi L- Filter are equal.

Conclution:

In this paper, we have defined anti fuzzy meet semi L-Filter, anti fuzzy level meet semi L-Filter and some theorems,lemmas & propositions were discussed.

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Selection of Single Sampling Plans by Variables using Pearson Type V Distribution

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ABSTRACT: *The procedures for constructing single sampling plans by variables based on Inverse Gamma distributions for known and unknown standard deviation for specified requirements to ensure protection to the producer and consumer are proposed and discussed.*

Keywords:

1.1 Single Sampling Plans by Variables Based on Inverse Gamma Distribution

The Pearson Type-V distribution is an Inverse Gamma Distribution and inverse Gamma distribution also called Inverted Gamma distribution.

Let X be a random variable representing a specific quality characteristic of manufactured items and be distributed according to a Inverse Gamma distribution with two parameters, namely, the shape parameter λ, and the scale parameter θ , The probability density function of the Inverse Gaussian distribution is given by

$$f(x,\lambda,\theta)=\frac{\theta^{\lambda}}{\Gamma(\lambda)}x^{-\lambda-1}\exp\left(\frac{-\theta}{x}\right), \quad x>0 \qquad (4.1.1)$$

The distribution function of the Inverse Gamma distribution is then defined by

$$F(x,\lambda,\theta)=\frac{\Gamma\left(\lambda,\frac{\theta}{x}\right)}{\Gamma(\lambda)}=Q\left(\lambda,\frac{\theta}{x}\right) \qquad (4.1.2)$$

Where the numerator is the upper incomplete gamma function and the denominator is the gamma function.

The basic properties of the Inverse Gamma distribution are defined in terms of the measures such as the mean , variance and skewness etc.,

$$mean=\frac{\theta}{\lambda-1} \quad \text{for } \lambda>1$$

$$\text{mod}e=\frac{\theta}{\lambda+1}$$

$$\text{var}\,iance=\frac{\theta^2}{(\lambda-1)^2(\lambda-2)} \quad \text{for } \lambda>2$$

$$skewnwess=\frac{4\sqrt{\lambda-2}}{\lambda-3} \quad \text{for } \lambda>3$$

$$kurtosis=3+\frac{30\lambda-66}{(\lambda-3)(\lambda-4)} \quad \text{for } \lambda>4$$

When the industrial production process exhibits a non-normal pattern with the measurements of the specific quality characteristic showing positively skewed and leptokurtic behavior, variables sampling plans can be defined based on a Inverse Gamma distribution. In this section, single sampling plans by variables based on the distribution are developed. A specific single sampling plan by variables can be defined under the following conditions:

(a) The quality characteristic, denoted by X, which is measurable on a continuous scale, is distributed according to a Inverse Gamma distribution with the shape parameter λ, and the scale parameter θ with the distribution function given by $F(x,\lambda,\theta)$.

(b) Each individual unit in a submitted lot is classified as a conforming or nonconforming unit according as $X \le U$ or $X > U$, where U is an upper specification limit. In the case of lower specification limit, L, an individual unit is classified as nonconforming if $X < L$.

A known standard deviation variables sampling plan for protection against the upper specification limit, according to Zimmer and Burr (1963), is operated using the procedure given below:

Step 1: Draw a random sample of n items from the population.

Step 2: Make measurements of the quality characteristic on these n items as $x_1, x_2, \ldots, x_n$ and compute the sample mean, $\overline{X}$.

Step 3: If $\overline{X} \le K$, then accept the lot; otherwise, reject the lot.

Thus, the sampling plan is designated by two parameters, namely, n, the sample size and K, the acceptability constant, which depends on the specification limit, U, the prescribed risks and the standard deviation, σ. The mathematical expressions for these parameters are derived based on the distribution function of the Inverse Gamma distribution and the related properties.

Let $F_x(x,\lambda,\theta)$ be the distribution function of X with mean, μ, and variance, σ^2. For a specified upper limit, U, the proportion, p, of nonconforming items, and the acceptance probability, $P_a(p)$, with reference to $F_x(x,\lambda,\theta)$ are defined from (4.1.1) and (4.1.2) as

$$p = P(X > U / \mu,\sigma) = 1 - F_x(U,\lambda,\theta) \qquad (4.1.3)$$

and $$P_a(p) = P\left(\overline{X} \le K \middle| \mu,\sigma\right). \qquad (4.1.4)$$

A procedure for designing a single sampling plan by variables is now developed for the requirement that the corresponding operating characteristic curve should pass through two designated points namely, $(p_1, 1-\alpha)$ and (p_2, β). The qualify levels p_1 and p_2 are defined from (4.1.3) by

$$p_1 = 1 - F_X(U,\mu_1,\sigma) \qquad (4.1.5)$$

and $$p_2 = 1 - F_X(U,\mu_2,\sigma), \qquad (4.1.6)$$

where μ_1 and μ_2 are the means of the underlying distribution corresponding to p_1 and p_2, respectively. The probability, α, of rejecting the lot having quality level p_1 and the probability, β, of accepting the lot having quality level p_2 are defined from (4.1.4) as

$$\alpha = 1 - F_X(K;\mu_1,\sigma/\sqrt{n}) = P(\overline{X} > K \mid \mu = \mu_1) \qquad (4.1.7)$$

and $$1 - \beta = P(\overline{X} > K \mid \mu = \mu_2). \qquad (4.1.8)$$

When the desired measures of skewness and kurtosis are specified, the parameter α and β can be determined from $\alpha_3 = \dfrac{4\sqrt{\lambda-2}}{\lambda-3}$ or from $\alpha_4 = 3 + \dfrac{30\lambda - 66}{(\lambda-3)(\lambda-4)}$

Let M and S be the mean and standard deviation of the Inverse Gamma distribution. Therefore, from (4.1.5) and (4.1.6), the quality levels p_1 and p_2 are defined as

$$p_1 = 1 - F(x_{p_1}, \lambda, \theta) = \frac{\Gamma\left(\lambda, \frac{\theta}{x_{p_1}}\right)}{\Gamma(\lambda)} = Q\left(\lambda, \frac{\theta}{x_{p_1}}\right) \quad (4.1.9)$$

$$p_2 = 1 - F(x_{p_2}, \lambda, \theta) = \frac{\Gamma\left(\lambda, \frac{\theta}{x_{p_2}}\right)}{\Gamma(\lambda)} = Q\left(\lambda, \frac{\theta}{x_{p_2}}\right) \quad (4.1.10)$$

Let us define the standardized variables $K^*_{p_1}$ and $K^*_{p_2}$, corresponding to p_1 and p_2, as

$$K^*_{p_1} = \frac{x_{p_1} - M}{S} \quad (4.1.11)$$

and $$K^*_{p_2} = \frac{x_{p_2} - M}{S}. \quad (4.1.12)$$

As the distribution of $\overline{X}$ is asymptotically normal, the probabilities α and 1-β are defined as area under normal curve with normal deviates K_α and $K_{1-\beta}$, respectively. Accordingly, when the upper specification limit, *U*, is specified, the expressions for *n* and *K* are, respectively, derived as

$$n_U = \left(\frac{K_\alpha + K_\beta}{K^*_{p_1} - K^*_{p_2}}\right)^2 = \left(\frac{S\left[K_\alpha + K_\beta\right]}{X_{p_1} - X_{p_2}}\right)^2 \quad (4.1.13)$$

and $$K = U - \frac{\sigma}{S}\left[\frac{K_\alpha x_{p_2} + K_\beta x_{p_1}}{K_\alpha + K_\beta} - M\right].$$

(4.1.14)

When the acceptance criterion is specified as $\bar{x} + k\sigma \le U$, the expression for *k*, according to Zimmer and Burr (1963), is given by

$$k_U = \left[\frac{K_\alpha x_{p_2} + K_\beta x_{p_1}}{S(K_\alpha + K_\beta)} - \frac{M}{S}\right]. \quad (4.1.15)$$

1.2 Determination of n and k of a Variables Sampling Plan

The parameters of a sampling plan by variables can be derived from a non-normal distribution when the third and fourth moment of the distribution of measurements are known or specified as $\alpha_3 = \frac{4\sqrt{\lambda - 2}}{\lambda - 3}$ and $\alpha_4 = 3 + \frac{30\lambda - 66}{(\lambda - 3)(\lambda - 4)}$.

In order to determine the required sampling plan by variables, the following procedure is followed:

Step1: Specify the measure of skewness (α_3) and kurtosis (α_4) of a Inverse Gamma distribution which has the shape parameter λ and scale parameter θ

Step 2: Specify the desired protection in terms of $(p_1, 1-\alpha)$ and (p_2, β).

Step 3: Determine the mean, *M*, and standard deviation, *S*, of the Inverse Gamma distribution as a function of λ and θ from the specified value of α_3 or α_4.

Step 4: Determine x_{p_1} and x_{p_2} from the cumulative distribution function of the Inverse Gamma distribution with the known value of λ and θ, satisfying equations (4.1.9) and (4.1.10), corresponding to the specified values of p_1 and p_2.

Step 5: Obtain $K^*_{p_1}$ and $K^*_{p_2}$ from equations (4.1.11) and (4.1.12) using *M* and *S*.

Step 6: Compute the normal deviates K_α and K_β for the specified values of α and β,

Step 7: Determine n_U and k_U using equation (4.1.13) and (4.1.15).

Based on the procedure described, the parameters, n_σ and k_σ, of the known standard deviation single sampling plans by variables are obtained for a wide range of values of p_1 and p_2 and tabulated in Tables 4.1.2 through 4.1.7 specifying the values of λ as 7, 12, 15, 20, 25 and 30 and β as fixed that is one. The parameters provided in the tables yield the maximum producer's risk of 5% and the maximum consumer's risk of 10%.

Example 1.1

It is required to determine a single sampling plan by variables in accordance with a desired discrimination when the measurable quality characteristic is distributed according to a Inverse Gamma distribution having the scale parameter $\lambda = 12$ and the shape parameter fixed at $\theta = 1$ The desired discrimination is specified to ensure protection to the producer and consumer in terms of quality levels $p_1 = 0.0075$ and $p_2 = 0.015$ associated with the producer's risk, $\alpha = 0.05$ and the consumer's risk, $\beta = 0.10$. As $\lambda = 12$ and $\theta = 1$ entering Table 4.1.2 with the specified quality levels, the parameters of the desired plan can be chosen as $n_\sigma = n_U = 23$ and $k_\sigma = k_U = 3.1581$.

Example 1.2

Suppose that a single sampling plan by variables is required to be instituted ensuring protection to the producer and the consumer. Assume that the quality characteristic under study follows a Inverse Gamma distribution which has the measure of skewness fixed at 1 and Measures of kurtosis fixed at 5. The desired protection is specified based on the producer's quality level fixed at 0.015 and the consumer's quality level fixed at 0.01, which are associated with 5% producer's risk and 10% consumer's risk. Entering Table 4.1.1 with the given measure of skewness and kurtosis , the value of $\lambda = 20$ and $\theta = 1$. Therefore, entering Table 4.1.5, with the producer's and consumer's quality levels, one obtains the parameters of the required plan as $n_\sigma = n_U = 96$ and $k_\sigma = k_U = 2.8635$.

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ON HARMONIOUS COLORING OF MIDDLE GRAPH OF SOME GRAPHS

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ABSTRACT: *A harmonious coloring is a proper vertex coloring in which each pair of colors appears on at most one pair of adjacent vertices. In this paper harmonious coloring of some graphs are studied . Some structural properties of them are discussed. Also, their harmonious chromatic number was obtained .*

Keywords: *belt graph, flower graph , middle graph, rose graph, harmonious coloring ,harmonious chromatic number.*
Classification number: 05C15, 05C75

1. Introduction

Let G be a finite, undirected graph with no loops and multiple edges. The graph G has the vertex set V (G) and the edge E (G).**Graph coloring** is coloring of G such that no two adjacent vertices share the same color.
A **harmonious coloring** [2, 3, 4, 6] is a proper vertex coloring in which every pair of colors appears on at most one pair of adjacent vertices. The harmonious chromatic number is the minimum number of colors needed for any harmonious coloring of G.
The **middle graph** of G denoted by M(G), is defined as follows :
The vertex set of M (G) is V(G) ∪E(G) in which two elements are adjacent in M(G) if the following conditions hold.
(i) x,y ∈ E(G) and x, y are adjacent in G.(ii) x ∈V(G), y ∈ E(G) and y is incident on x in G.
A belt graph is obtained from the helm graph by joining its outer vertices with 2n+1 vertices and 4n edges.
A flower graph is obtained from a cycle C by joining two consecutive vertices to a new vertices. It has 2n vertices and 3n edges.
A rose graph is obtained from a wheel graph (n≥4) by joining the two consecutive vertices to a new vertices.

2. Structural properties of middle graph of Belt, Flower and Rose graphs

- Number of vertices in M (B_n), p= 6n+1
- Number of vertices in M[(F_n)] ,p=5n
- Number of vertices in M[R_n)] ,p=6n+1
- Maximum degree in M (B_n), Δ=2n+2
- Maximum degree in M[(F_n)], Δ=2n-1
- Maximum degree in M (R_n), Δ=2n
- Minimum degree in M(B_n), δ =2
- Minimum degree in M[(F_n)], δ = 2
- Minimum degree in M[R_n], δ = 2

3. Harmonious coloring of M (B_n) ,M [(F_n)] and M(R_n)

Theorem 3.1 :
For the middle graph of M(B_n), the harmonious chromatic number is 5n-1, n≥3
(i. e) χ_H [M (B_n)] = 5n-1
Proof:
Let {v_1,v_2……v_n, u_1,u_2……u_n ,w_1} be the vertices of the Belt graph B_n
Let G =M(B_n) be the middle graph of belt graph.
Then by the definition of middle graph ,each edge is sub-divided by a new vertex.
The edges (u_i,u_{i+1}) and (u_n , u_1) is obtained from the outer vertices of the helm graph are joined.
Assume that each edge (u_i,u_{i+1}) and (u_n , u_1) are sub-divided by the new vertex x_i , y_i, z_i ,e_{jj} ,i&j=1 to n respectively.
Now the coloring assignment are as follows

The vertices $\{v_1,v_2.....v_n\}$ are colored with the colors $c_1,c_2.....c_n$ and the vertices $\{u_1,u_2.....u_n\}$ are colored with the colors $c_n, c_{n+1}, c_{n+2},.... c_{2n-1}$

The new vertex $\{x_1,x_2,.....x_n\}$are colored with the colors $c_{2n+1}, c_{2n+2}, ,.... c_{3n}$ and the vertex $\{y_1,y_2,.....y_n\}$ are colored with the colors $c_{2n-1}, c_{2n},.... c_{3n-2}$

The vertices $\{z_1,z_2,.....z_n\}$ are colored with the colors $c_{2n-1}, c_{2n},.... c_{3n-2}$

Atlast the vertex$\{e_{11},e_{22}.....e_{nn}\}$are colored with the colors $c_{4n-1}, c_{4n}.....c_{5n-2}$

The root vertex w_1 is colored with the color c_{5n-1}

Clearly, the above said coloring is harmonious and is minimum.

Therefore, $\chi_H [M (T_n)] = 5n-1$

Fig 1: Middle graph of Belt graph

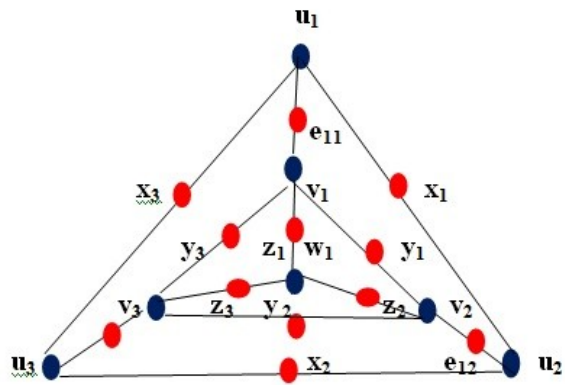
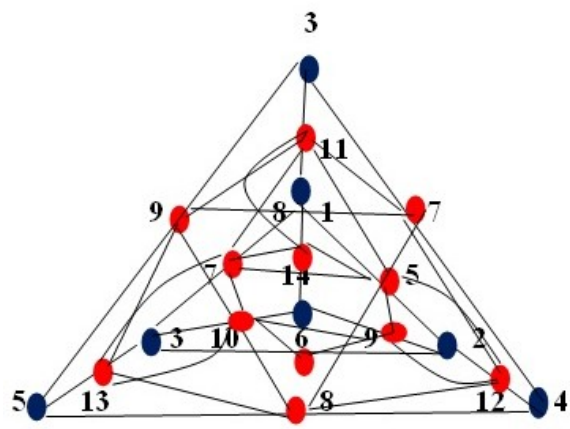


Fig 2: Middle graph of Belt graph B3



$\chi_H [M (T_3)] = 5n-1 = 14$

Theorem 3.2 :

For the middle graph of $M(F_n)$, the harmonious chromatic number is 5n-2, n≥3

(i. e) $\chi_H [M (F_n)] = 5n-2$

Proof:

Let F_n be the flower graph with 2n vertices and 3n edges.

Let $\{v_1,v_2.....v_n, u_1,u_2.....u_n,\}$ be the vertices of the Flower graph F_n

Then by the definition of middle graph , each edge is sub-divided by a new vertex.

Therefore, assume that each edge (v_i,v_{i+1}) and the line joining v_i and v_{i+1} to a new vertex ui ,i=1 to n are sub-divided by the new vertices x_i , y_i, e_{jj} ,i&j=1 to n respectively.

Now the coloring assignment are as follows

The vertices $\{v_1,v_2.....v_n\}$ are colored with the colors $c_1,c_2.....c_n$ and the vertices $\{u_1,u_2.....u_n\}$ are colored with the colors $c_n, c_{n+1}, c_{n+2},.... c_{2n-1}$

The sub-divided vertex$\{e_{11},e_{22}.....e_{nn}\}$are colored with the colors $c_{4n-1}, c_{4n}.....c_{5n-2}$ and $\{y_1,y_2,.....y_n\}$ are colored with the colors $c_{3n-1}, c_{3n},.... c_{4n-2}$

Atlast, the sub-divided vertex $\{x_1,x_2,\ldots\ldots x_n\}$are colored with the colors $c_{2n-1}, c_{2n}, \ldots c_{2n+1}$ and
Clearly, the above said coloring is harmonious and is minimum.
Therefore, $\chi_H [M(F_n)] = 5n-2$

Fig 3: Middle graph of Flower graph

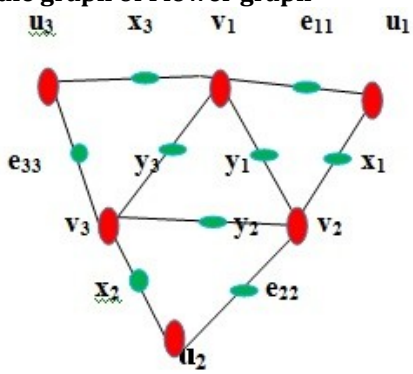
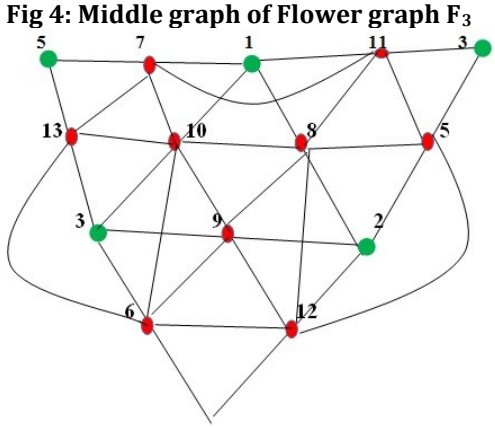


Fig 4: Middle graph of Flower graph F_3



Theorem 3.2 :
For the middle graph of $M(R_n)$, the harmonious chromatic number is $4n+2$, $n\geq 4$
(i. e) $\chi_H [M(R_n)] = 4n+2$

Proof:
Let R_n be the rose graph with 2n+1 vertices and 4n edges.
Let $\{v_1,v_2\ldots\ldots v_n, u_1,u_2\ldots\ldots u_n, w_1\}$ be the vertices of the Rose graph R_n
The neighbours of v_i and v_{i+1} are represented by y_i ,i=1 to n and are colored with the colors $c_{2n+1}, c_{2n+2}, \ldots c_{3n}$.
The outer edges are sub-divided by z_i ,i=1 to n and are and are colored with the colors $c_{n+1}, c_{n+2}, \ldots c_{2n}$.
The edges $\{e_{11},e_{22}\ldots\ldots e_{nn}\}$are colored with the colors $c_{3n+2}, c_{3n+3}\ldots c_{4n+1}$
Atlast,the vertex w_1 is colored with the color c_{4n+2}
Clearly, the above said coloring is harmonious and is minimum.
Therefore, $\chi_H [M(R_n)] = 4n+2$

Fig 5: Middle graph of Rose graph

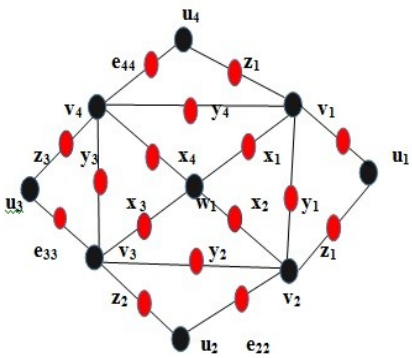
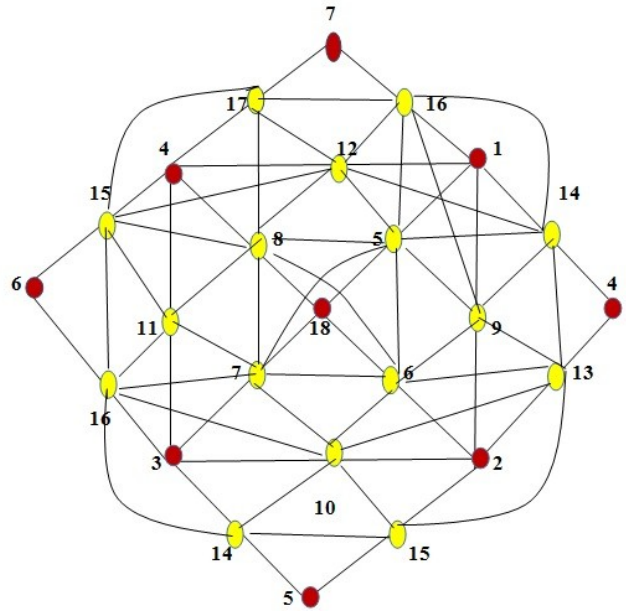


Fig 6: Middle graph of Rose graph R_4



χ_H [M (R_4)] = 4n+2=18

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Classification of Breast cancer Using Machine Learning Techniques

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ABSTRACT: *A disease that is commonly misdiagnosis is breast cancer. Earlier diagnosis of breast cancer saves enormous lives failing which lead to other may severe problems causing sudden fatal end. Breast cancer classification can be done by using machine learning techniques. Machine Learning (ML) is a field that gives computer ability to learn without explicitly programmed. Machine Learning (ML) techniques are being broadly used in the breast cancer classification problem. They provide high classification accuracy and effective diagnostic capabilities. In this paper, many different types of machine learning classification algorithms like Linear Regression, Decision Tree, Multi-Layer Perceptron and K-Nearest Neighbor were implemented data classify Wisconsin Breast Cancer Data set that is obtained from the UCI Machine, Learning Repository, an online open source repository This data set was collected periodically over three years by Dr. William H. Walberg from the University of Wisconsin Hospitals and consists of 569 instances. Classification of the unseen data using different k for k-fold crosses validation and accuracy.*

Keywords: *Breast Cancer, Classification, Machine Learning, Linear Regression, Decision Tree, Multi-Layer Perceptron and K-Nearest Neighbor.*

I. Introduction

Breast cancer (BC) is one of the most common cancers in females worldwide, representing the majority of new cancer cases leading to related death according to global statistics. The early detection and diagnosis of breast cancer can improve the prognosis and chances of survival significantly as it can promote timely clinical treatment to patients. Machine learning and classification algorithms are an effective way to classify data. In medical field these methods are used in detect, diagnosis and analysis to make decisions.

Mammography is important screening test for breast cancer and it is an X-ray of the breast. It can detect and diagnose breast cancer up to two years before tumor can be felt by the patient or a doctor. There are two kinds of mammography are used. They are screen-film mammography (SFM) and digital mammography (DM). SFM is utilized in asymptomatic ladies breast i.e. problem free breast where it receipts two sights of both breasts. The time duration of screening mammograms (conventional mammography) is around 20 minute. It cannot detect benign cancer properly. Digital Mammography overcomes the problem of screening mammograms. It is related with computer system i.e. the data of digital mammography are kept in a computer. In this, the images are taken and image processing methods are applied to improve the quality of the image. DM performs better in case of misdiagnosed cancer samples. Magnetic Reasoning Imaging (MRI) is another most common technique for the diagnosis of breast cancer [2]. Women age 40-45 or older who are at average risk of breast cancer should have a mammogram once a yea , women at high risk should have yearly mammogram along with an MRI starting at age 30. The chance of getting breast cancer increases as women become older. Nearly 80% of breast cancer found in women over the age of 50. Women who had breast cancer in one breast at an increased risk of developing cancer in her other breast also. A woman has higher risk of breast cancer if her mother, sister or daughter had breast cancer, especially at young age. Women with certain genetic mutations including changes to the BRCA1 and BRCA2 genes are at higher risk of developing breast cancer during their lifetime.

However several techniques have been presented, are able to provide an accurate and reliable outcome. Those techniques can operate without any medical equipment's and medical staffs. We introduce an innovative modality to classify the input attributes according to the presence or absence of benign or malignant types of breast cancer [3].

In this paper, four machine learning techniques are applied to a breast cancer data set. The data

set is trained, tested and compared. The techniques applied are Linear Regression (LR), Decision Tree (DT), Multi-Layer Perceptron (MLP) and K-Nearest Neighbor (KNN).

II. LITERATURE REVIEW

Variety of breast cancer screening modalities including mammography, ultrasound, CT scan, MRI, PET, radar-based microwave imaging, RF sensors, biosensors, arrays, biopsy and biomarker detection in DNA and genes are existing[4]. Different classification techniques were developed for breast cancer diagnosis, the accuracy of many of them was evaluated using the dataset taken from Wisconsin breast cancer database[5 In cancer classification, KNN can be used to measure the performance of false positive rates [6]. The Author Moh'dRasoul used DWT tool for image filtering and BPNN for processing and achieved 93.7% accuracy [7]. The author Clemen Deng used WHAVE algorithm with Wisconsin Breast Cancer Database and achieved 99% accuracy [8]. The author Ashwaq Qasem used marker Controller Watershed algorithm and achieved 95% accuracy[9] The author AlirezaIsareh used signal to noise ratio with SVM and achieved 98.80% accuracy [10]. The author Junaid Ahmad used Adaptive Resonance Theory with UCI database and got 84.21% accuracy [11]. By B.M. Gayathri work on comparative study about Relevance vector machine and achieved 97% accuracy [12].The author Ms.H.R. Mhaske used KNN and SVN with 150 images database and achieved 80-90% accuracy [13]. The author S. Aruna used Naïve Bayes and SVM with UCI database and achieved 68-79% accuracy[14]. Yohannes Tsehay developed a weakly supervised computer aided detection system that was used in biopsy to learn data[15].

III. METHODOLOGY

Machine learning

Machine Learning is a concept which allows the machine to learn from examples and experience, and that too without being explicitly programmed. Types of Machine Learning [24].

Machine learning is sub-categorized to three types

- Supervised Learning
- Unsupervised Learning
- Reinforcement Learning

Supervised learning

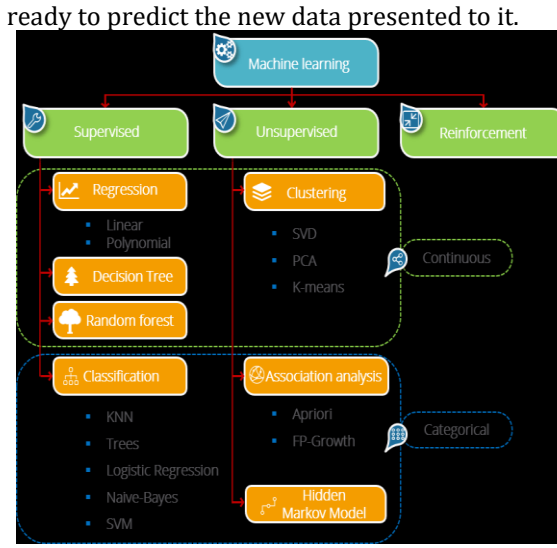
Supervised Learning is the one, where you can consider the learning is guided by a teacher. We have a dataset which acts as a teacher and its role is to train the model or the machine. Once the model gets trained it can start making a prediction or decision when new data is given to it.

Unsupervised learning

The model learns through observation and finds structures in the data. Once the model is given a dataset, it automatically finds patterns and relationships in the dataset by creating clusters in it. What it cannot do is add labels to the cluster; like it cannot say this a group of apples or mangoes, but it will separate all the apples from mangoes.

Reinforcement learning

It is the ability of an agent to interact with the environment and find out what is the best outcome. It follows the concept of hit and trial method. The agent is rewarded or penalized with a point for a correct or a wrong answer, and on the basis of the positive reward points gained the model trains itself. And again once trained it gets ready to predict the new data presented to it.



Classification

In machine learning and statistics, classification is a supervised learning approach in which the computer program learns from the data input given to it and then uses this learning to classify new observation. Here we have the types of classification algorithms in Machine Learning[16]. Are described below Experiments have been done in these classifiers with Wisconsin Breast Cancer Database.

Linear Regression

Linear regression is used for finding linear relationship between target and one or more predictors. There are two types of linear regression- Simple and Multiple. Here the output label being classifications is benign or malignant (B or M) like all other regression models, it is used for relationship between independent and dependent variables. The variable we are predicting is called the criterion variable and is referred to as Y. The variable on which predictions

are made is called the predictor variable and is referred to as X Linear regression. It consists of finding the best-fitting straight line through the points and the best-fitting line is called a regression line.

Decision Tree

Decision Tree (DT) is a supervised Machine Learning (ML) method used for classification and regression analysis. Decision tree is based on the divide and conquers methodology. It divides the partition by Numerical partition method and Nominal attributes methods. In Numerical partitions, partitions are formed on the basis of discrete values with some conditions and in Nominal partition, the partitions are formed on the basis of nominal attributes. It leads splitting of the tree depending on attributes values. In the research carried out, C4.5 decision tree algorithm is used for experimental analysis. This technique generates a tree that can have categorical as well as discrete attributes. Accuracy of each rule generated is estimated to determine the order in which attribute will be placed in the decision tree. The most common approach is to minimize the average error of every node

Multi-Layer Perceptron

Artificial neural networks are derived from biological neural networks and are capable of approximating any functions with any accuracy that is desired. In addition, they are very robust and adaptive against uncertainties. To make use of these benefits, designers should adjust some parameters such as the number of hidden layers and the number of neurons in the hidden layer. However, this method is a black-box approach, which means we cannot perceive the internal structure that leads to a good response. The neural network consists of an input layer that includes input features, weight matrixes ($W1$ and $W2$) that make connections between the layers, the hidden layer which includes activation functions that are usually the sigmoid or logistic function, and an output layer that can be a nonlinear or linear function.[17] They need to train favorable results since they are supervised networks. Because this classifier learns to convert input data into a favorable result, they are broadly applied for pattern classification [18].Multilayer Perceptions have been applied successfully to solve some difficult and diverse problems by training them in a supervised manner with a highly popular algorithm known as the error back-propagation algorithm [19].

K-Nearest Neighbor

K-Nearest Neighbor technique is a non-parametric approach used for Regression and classification analysis. The input comprises of k, K is a number of classes in dataset, and it is a small and positive integer. Any input data is classified and distinguished by the majority of its neighbors. So, the input data is allotted to the class which is higher in its K nearest neighbors. In this research work the value of k = 2, then the input data will be assigned to a class of two nearest neighbor, since the dataset contain only two classes. The classes are cancerous tumor and non-cancerous tumor. Euclidean distance is used to measure the distance between two data point [20]. Given N training vector, suppose we have A and Z as training vectors in this bidimensional features space, we want to classify c which is feature vector [21].

Diagnosis of breast cancer is an arduous task. The first step is the clinical examination to detect the tumor in the breast by the General Surgeon or by using imaging techniques such as Mammography. This is followed by FNA on the lump detected. The FNA procedure provides with attributes such as tumor size, radius, area etc. These can be used as features for modelling the data into a classifier. Finally, models such as linear regression, k-nearest neighbor, decision tree and multi-layer Perceptron are trained using the obtained data to make predictions.

IV. EXPERIMENTAL RESULTS

Our investigation is based on the Original Wisconsin Breast Cancer Data set that is obtained from the UCI Machine, Learning Repository, an online open source repository [16]. This data set was collected periodically over three years by Dr. William H. Wolberg from the University of Wisconsin Hospitals and consists of 569 instances. Classification of the unseen data using different k for k-fold crosses validation. For this purpose 30%, 40%, 50% of data is considered as unseen data and they are tested based on the model file related to train set. In this study, 10-fold, 5-fold and 3-fold cross validation are used to separate the datasets into two categories training and testing. The benefit of using k-fold cross validation is to test the entire dataset [22]. K-Fold cross validation improves upon the validation set approach by dividing the n observations in to k mutually exclusive , and approximately equally sized subsets known as “folds” the first folds becomes a validation set while the remaining k-1 fold become the training set . The model is fit on training set and its test error is estimated on the validation set [23]. The implementation done by using python (3.7) –IDE called spyder. The attributes used for classification is as tabulated as follows.

TABLE I. WISCONSIN DIAGNOSIC BREAST CANCER DATASETDESCRIPTION

S no.	Attribute/Feature	Range
1	ID Number Identification	number for patients
2	Diagnosis	2: Benign 4:Malignant
3	Radius	11-27
4	Area	360-2300
5	Perimeter	71-82
6	Texture	11-40
7	Smoothness	0.05-0.2
8	Compactness	0.04-0.45
9	Concavity	0.02-0.5
10	Concave Points	0.02-
11	Symmetry	0.1-0.3
12	Fractal Dimension	0.05-0.1

The dataset contain 569 rows and 32 columns. Diagnosis is a column in which we are going to classification when cancer is (M) this mean is malignant or(B) is benign the training and testing split for modeling of data is performed by save 70% for training and 30% for testing.

This section presents the performance of the classifiers on the Original Wisconsin Breast Cancer Data set that is obtained from the UCI Machine, Learning Repository, an online open source repository the dataset obtained from human patients. All algorithms were implemented in spyder (python). We applied the cross-validation method with leave-one-out technique to estimate the sensitivity and specificity. Table 2 shows the sensitivity and specificity results for the classification of tumors for the pilot dataset of 569 patients.

Table 2 shows that for dataset, the most accurate result for linear regression, Decision Tree, Multi-Layer Perceptron and K-Nearest Neighbor

Classifiers	Average accuracy with 10-fold cross validation	Average accuracy with 5- fold cross validation	Average accuracy with 3-fold cross validation
LR	98.2%	98.5%	98.8%
DT	94.5%	98.7%	95.8%
MLP	98.7%	92.3%	92.6%
KNN	97.6%	98.6%	96.08

Shown in table 1 for WBC dataset, the most accurate result for LR belongs to k=3. Likewise, the more accurate result for DT and KNN belongs to k= 5. Also, for MLP, the more accurate result is for k=10

Fig 1 linear Regression

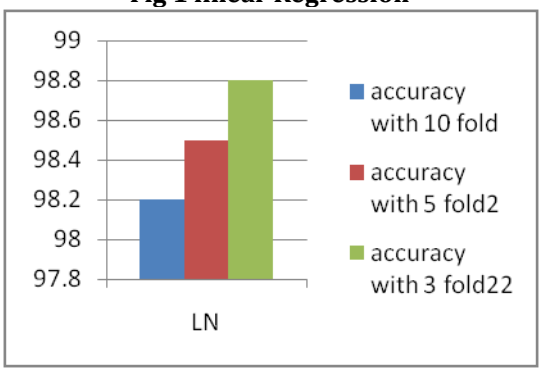


Fig 2 Decision Tree

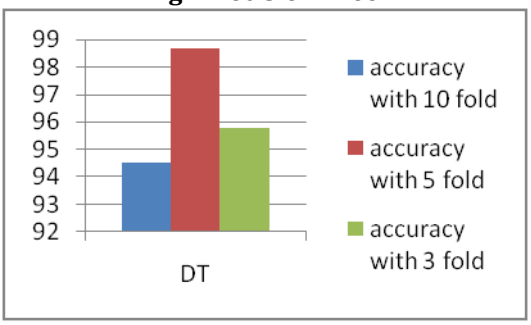


Fig 3Multi-Layer Perceptron

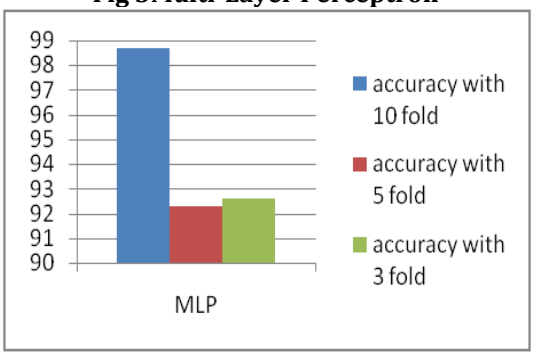
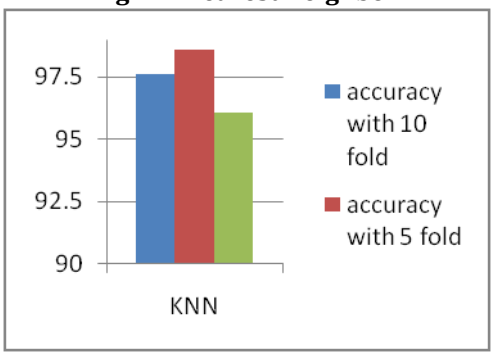


Fig 4 K-Nearest Neighbor



V.CONCLUSION

On the Wisconsin Breast Cancer datasets, we used four algorithms, which are: LR, DT, MLP and KNN, the true accuracy of a give model is cross

validation which is based on splitting the available dataset into a training set and a validation set. Breast cancer classification is to build classifiers that are precise and reliable. After an accurate comparison between our algorithms, we noticed that LR achieved a higher efficiency of 98.8%,

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COMPUTING PATH SUM CONNECTIVITY INDEX OF DENDRIMER NANOSTARS

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ABSTRACT: *Topological indices are used to define chemical molecular structures which are used in the study of QSAR/QSPR. There are many applications of graph theory in the field of structural chemistry. In this paper we found exact general (α, 2) - path sum connectivity index and the monotonicity of Dendrimer Nanostars.*

Keywords: *Dendrimer Nanostar , Molecular graph , Topological indices .*
AMS Subject classification: *05c90, 05c07, 05c12, 05c31.*

Introduction

In this work G is a simple graph with set of vertices and edges. Here vertices represents atoms and edges represents bonds and we denote degree by d(u) and d(v)for the vertices u and v respectively. The topological descriptors or topological indices are defined to describe the physico chemical properties of chemical compounds or network structures .different index represents its corresponding chemical structures in graph theoretical terms via arbitrary molecular graph.

Numerous studies reveals that there is strong natural relationship between the chemical characteristics of chemical compounds and drugs for these reasons we are using many two dimensional topological indices which is essential approach to find many new drugs. Further improvement in the study of chemical indices three dimensional molecular features and molecular chirality are also discovered.

The general (α ,t) path sum connectivity index of a molecular graph arise from three dimensional quantitative structure – activity relationships (3D QSAR) and molecular chirality. It is defined as $t_{\chi_\alpha}(G) = \sum_{p^t = v_{i0}v_{i1}\dots v_{it}} [d_G(v_{i0}) + d_G(v_{i1}) + \cdots + d_G(v_{it})]$ with any non zero real no α and t. According to this definition the general (α,t) path sum connectivity index of an arbitrary graph is an important invariant under graph automorphism.

Dendrimer

Dendrimers are repetitively branched molecules. The name comes from the greek word “dendron”. A dendrimer is typically symmetric around the core and ogten adopts a spherical three dimensional morphology. The word Dendron is also encounterd frequently. A Dendron usually contains a single chemically addressable group called the focal point or core. Dendrimers are function as detecting agents because their structure can lead to multivalent system .It can also be used as solubilizing agent. These are considered one of the most important commercially available building blocks in nanotechnology. Its diameter increases linearly where as the number of surface groups increases geometrically .Dendrimers are very uniform with dimensions incrementally grown in approximate nanometer steps from 1 to 10 nm. The control over size ,shape and surface functionally makes dendrimers are of the commercially available smartest nanotechnologies[6].

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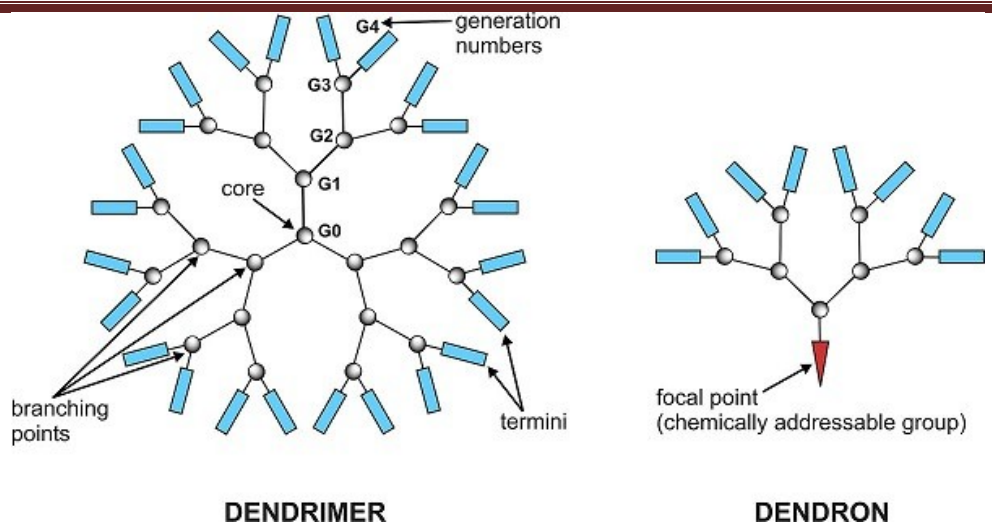
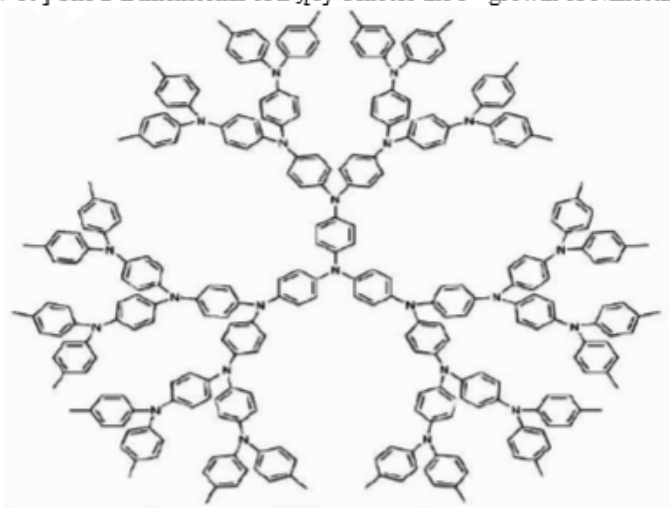


Structural properties of Dendrimer Nanostar :

Figure 1. [37-39] The 2-Dimensional of $D_3[3]$ denotes the 3th growth of Nanostar Dendrimer.

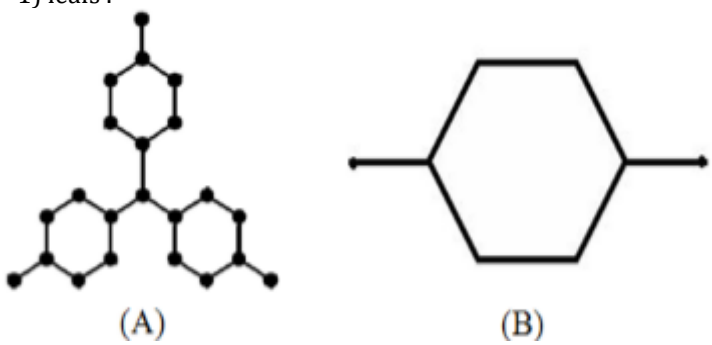


(i)Using structural representation of dendrimer nanostar $D_3[n]$ the set of vertices and edges are calculated in [4].

The number of vertices = $21(2^{n+1}) - 20$

The number of edges $=24(2^{n+1} -1)$

(ii)Here $D_3[0]$ is called core(fig A) and $D_3[n]$ created by $3(2^n)$ leafs(fig B) to $D_3[n-1]$ in n th growth. Therefore $D_3[n]$ have totally $3(2^{n+1}-1)$ leafs .



General $(\alpha, 2)$ path sum connectivity index of Dendrimer nanostar $D_3[n]$

The classification of components of dendrimer structure is clearly derived in [3].

Theorem 1: For an arbitrary real number α ,the general (α,2)- path sum connectivity index of Dendrimer Nanostar is $2_{\chi_\alpha}(D3[n]) = 3\ (2^{n+1} - 1)\ [4.8^\alpha + 6.7^\alpha + 3.3^\alpha] + 3.9^\alpha$.

Proof : Let $v_{i0}v_{i1} \dots v_{it}$ be a path p^t of length t in dendrimer nanostar and $d_G(v_{i0}), d_G(v_{i1}) \dots d_G(v_{it})$ is called its degree sequence. Seeing the structure of two dimensional dendrimer nanostar $D_3[n]$, there are five types of degree sequence having five types of path of length two (i.e [(3,3,3),(3,3,2),(3,2,2),(2,2,3)and(2,3,2)]).
Let m_{ijk} denotes the number all two paths of the degree sequence types (i,j,k) in $D_3[n]$ where i,j and k are positive integers.
We computed the number of paths of degree 2 as follows,

Degree sequence	No of paths
(3,3,2)	$12(2^{n+1}-1)$
(3,2,2)	$6(2^{n+1}-1)$
(2,2,3)	$6(2^{n+1}-1)$
(2,3,2)	$6(2^{n+1}-1)$
(3,3,3)	$3+9(2^{n+1}-1)$

$m_{332} = 12(2^{n+1}-1)$, $m_{322} = 6(2^{n+1}-1)$, $m_{223} = 6(2^{n+1}-1)$, $m_{232} = 6(2^{n+1}-1)$,
$m_{333} = 3+9(2^{n+1}-1)$.
Let us consider $2_{\chi_\alpha}(G) = \sum_{p^2=v_{i0}v_{i1}v_{i2}}[d_G(v_{i0}) + d_G(v_{i1}) + d_G(v_{i2})]$

$$= m_{332}(3+3+2)^\alpha + m_{322}(3+2+2)^\alpha + m_{223}(2+2+3)^\alpha + m_{232}(2+3+2)^\alpha + m_{333}(3+3+3)^\alpha$$

$$= 12(2^{n+1}-1)\ 8^\alpha + 6(2^{n+1}-1)\ 7^\alpha + 6(2^{n+1}-1)\ 7^\alpha + 6(2^{n+1}-1)\ 7^\alpha + [3+9(2^{n+1}-1)]\ 9^\alpha$$

$$= 3\ (2^{n+1}-1)\ [4.8^\alpha + 6.7^\alpha + 3.3^\alpha] + 3.9^\alpha$$

Theorem 2: Let $D_3[n]$ be the general representation of the class of dendrimer nanostar shown in figure 1. Then
(a) For any real number α,we have $2_{\chi_\alpha}(D3[n])$is strictly increasing with respect to all positive integer n.
(b) The smallest general (α,2)- path sum connectivity index of Dendrimer Nanostar is

$$2_{\chi_\alpha}(D3[n])_{min} = 3\ \{[4.8^\alpha + 6.7^\alpha + 3.3^\alpha] + 9^\alpha\} \quad \text{if and only if } n=0$$

which implies that it is a core .

Conclusion

The Dendrimer can have promising future in various pharmaceutical and diagnostic fields as they possess distinct properties such as multivalence , high degree of branching and well defined molecular weight. The advancement in synthesis of dendrimers helps to produce different structure. Hence the increased research in the field of dendrimers its utility for wide range of application in drug delivery systems.

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State Dependent Bulk Arrival Multi-Stage Retrial G-Queue with Feedback and Vacation

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ABSTRACT: *M^X/ G/ 1 retrial G-queue with feedback and vacation is considered. The server provides M stages of heterogeneous service in succession. Admission of each individual customer to the system depends upon the state of the server. Positive customers arrive in batches according to Poisson process. Arrival of a negative customer removes the positive customer being in service and makes the server down. If the server is idle upon the arrival of a batch, one of the customers in the batch commences the first stage of service and the rest join the orbit. After the completion of the first stage, the customer may proceed to the second stage with probability θ_1 , feedback to the orbit with probability δ_1 or departs the system with probability q_1 (= $1 - \theta_1 - \delta_1$) and so on. At the final stage, the customer will feedback to the orbit with probability δ_M or depart the system with probability q_M (= $1 - \delta_M$). The server takes random vacation with probability τ . The steady state analysis and probability generating functions of the system are obtained by using the supplementary variable technique and some performance measures are derived.*

Keywords:

1.Introduction

The server vacation play a major role in retrial queueing models such as communication networks, computer, production and manufacturing systems. Review of retrial queueing literature can be found in the survey of Kim and Kim (2015). The name G-queue refers the queue with negative customers. Arrival of negative customers removes the customers from the system and makes the server down like virus has the potential to cause unexpected or damaging effects, such as harming the system software by corrupting or destroying data. Gelenbe (1989) was first introduced this type of queue. Yang et al. (2013) analyzed $M^X/G/1$ retrial G-queue with single vacation by using supplementary variable technique. Gao and Wang (2014) analysed an M/G/1 G-queue with Orbital search and non-persistent customers. Sumitha and Udaya chandrika (2015) analyzed the batch arrival retrial queue with positive and Negative customers, Priority or Collisions, Delayed Repair and Orbital search. Rajadurai et al. (2015) studied an $M^X/G/1$ unreliable retrial G-queue with Orbital search, feedback and Bernoulli vacation. Recently, Palani, Chandrasekaran and Indhira (2017) derived state dependent arrival in bulk retrial queueing system with immediate Bernoulli feedback, Multiple vacations and threshold. In this paper, state dependent bulk arrival Multi-stage retrial G-queue with feedback and vacation is considered.

2. Model Description

Arrival Process

Negative customers arrive according to the Poisson process with rate λ^- . The positive customers arriving in batches according to Poisson process with rate λ^+. The batch size Y is a random variable with distribution P (Y = k) = C_k, k = 1, 2, ... Let C(z) be the generating function of the batch size distribution with first two moments m_1 and m_2. Let χ_1, χ_2, χ_3 and χ_4 be the probabilities for the admitting a customer when the server is idle, busy, repair and on vacation respectively. Then the corresponding probabilities of admitting n customers from the arriving batch of k customers is given by

$$a_{j,n} = \begin{cases} \sum_{k=1}^{\infty} C_k (1-\chi_j)^k , & n=0, \quad j=1,2,3,4 \\ \sum_{k=1}^{\infty} C_k \binom{k}{n} \chi_j^n (1-\chi_j)^{k-n} , & n \geq 1, \quad j=1,2,3,4 \end{cases}$$

Let $\lambda_j = \lambda \sum_{n=1}^{\infty} a_{j,n} \quad for \quad j=1,2,3,4$

The probability generating function $a_j(z)$ of the sequence $\{a_{j,n}, n \geq 0\}$ is given by $a_j(z) = C(\chi_j z + (1-\chi_j))$ with first two moments $\chi_j m_1$ and $\chi_j^2 m_2$, j = 1,2,3,4

Retrial Process

If an arriving batch finds the server either busy or on vacation or breakdown, then the batch joins into an orbit. If the server is free, the first stage service commences for one of the arriving customers and others join the orbit. Inter-retrial times form an arbitrary distribution A(x) with corresponding Laplace-Stieltjes transform A*(s). and the hazard rate function $\eta(x) = \dfrac{dA(x)}{1-A(x)}$.

Service Process and Feedback Rule

The server provides M stages of heterogeneous service in succession. After the completion of the first stage service, the customer may proceed to second stage with probability θ_1, feedback to the retrial group with probability δ_1 or depart the system with probability q_1 (= $1 - \theta_1 - \delta_1$). After completing 2nd stage service, the customer may opt 3rd stage with probability θ_2, feedback to the retrial group with probability δ_2 or depart the system with probability q_2 (= $1 - \theta_2 - \delta_2$). This process continues upto M stages. After final stage service the customer may feedback to the retrial group with probability δ_M or depart the system with probability q_M (= $1 - \delta_M$).

The service times are generally distributed with distribution function $B_i(x)$ and LST $B_i^*(s)$ with first two moments $\mu_i^{(1)}$ and $\mu_i^{(2)}$ and the hazard rate function $\mu_i(x) = \dfrac{dB_i(x)}{1-B_i(x)}, \; i = 1,2,....,M$.

Define $\Lambda_i^* = B_1^* B_2^* B_3^* ..... B_i^* \quad with \quad \Lambda_0^* = 1 \quad and \quad Q_{i-1} = \theta_1 \theta_2 \theta_3 ..... \theta_{i-1} \quad with \; Q_0 = 1$. The first moment $M_{1,i}$ and the second moment $M_{2,i}$ are given by

$$M_{1,i} = \lim_{z \to 1} \Lambda_i^{*\prime}(g(z)) = \sum_{j=1}^{i} \left[B_j^{*\prime}(\lambda^-) \prod_{\substack{k=1 \\ k \neq j}}^{i} B_k^*(\lambda^-) \right]$$

$$M_{2,i} = \lim_{z \to 1} \Lambda_i^{*\prime\prime}(g(z)) = \sum_{j=1}^{i} \left[B_j^{*\prime\prime}(\lambda^-) \prod_{\substack{k=1 \\ k \neq j}}^{i} B_k^*(\lambda^-) \right] + 2 \sum_{j=1}^{i-1} \left[B_j^{*\prime}(\lambda^-) B_{j+1}^{*\prime}(\lambda^-) \prod_{\substack{k=1 \\ k \neq j+1}}^{i} B_k^*(\lambda^-) \right]$$

where $g(z) = \lambda^+ + \lambda^- - \lambda^+ a_2(z)$

Removal Rule and Repair process

The arrival of a negative customer eliminates the positive customer being in service from the system and makes the server down. The repair of the failed server commences immediately. The repair time of the server failed during i^{th} stage follows general distribution with distribution function $R_i(x)$ and LST $R_i^*(s)$ for (i=1,2,...M) with first two moments $\beta_{i,1}$ and $\beta_{i,2}$ respectively and the hazard rate function $\beta_i(x) = \dfrac{dR_i(x)}{1-R_i(x)}, \; i = 1,2,....,M$.

Vacation process

The server may take a random vacation with probability τ or remain in the system with probability $\bar{\tau}$. The vacation times are distributed with distribution function $V_i(x)$ and LST $V_i^*(s)$ for (i=1,2,...M) with first two moments $\gamma_{i,1}$ and $\gamma_{i,2}$ respectively and the hazard rate function $\gamma_i(x) = \dfrac{dV_i(x)}{1-V_i(x)}$, $i = 1,2,....,M$.

3. Stability Condition

Let $\{t_n\ ,\ n \in N\}$ be the epoch of the service completion or a vacation completion or the repair period ends. The sequence of random variables $Y_n = \{C(t_n^+), X(t_n^+)\}$ forms an embedded Markov chain which is ergodic iff

$$\lambda^+ / \lambda_1^+ \chi_1 m_1 \left(1 - A^*(\lambda_1^+)\right) + \sum_{i=1}^{M} \delta_i Q_{i-1} \Lambda_i^*(\lambda^-) + \sum_{i=1}^{M} (q_i + \delta_i) Q_{i-1} M_{i,1} + \sum_{i=1}^{M} Q_{i-1} M_{i-1,1} \overline{B}_i^*(\lambda^-)$$
$$+ \sum_{i=1}^{M} Q_{i-1} M_{i,1} \Lambda_{i-1}^*(\lambda^-) + \lambda^+ \chi_2 m_1 \sum_{i=1}^{M} Q_{i-1} \Lambda_{i-1}^*(\lambda^-) \overline{B}_i^*(\lambda^-) / \lambda^-$$
$$+ \chi_3 m_1 \sum_{i=1}^{M} Q_{i-1} \Lambda_{i-1}^*(\lambda^-) \overline{B}_i^*(\lambda^-) \beta_{i,1} + \tau \chi_4 m_1 \sum_{i=1}^{M} (q_i + \delta_i) Q_{i-1} \Lambda_i^*(\lambda^-) \gamma_{i,1} < 1$$

4. Steady State Distribution

Define the state of the server as

$$C(t) = \begin{cases} 0\ , & \text{the server is idle} \\ i\ , & \text{the server is busy at } i^{th} \text{ stage} \\ M+i\ , & \text{the server is repair at } i^{th} \text{ stage} \\ 2M+i\ , & \text{the server is on vacation at } i^{th} \text{ stage} \end{cases}$$

For the process, {N(t) ; t ≥ 0}, define the probabilities

$I_0(t)$ = P{C(t) = 0, X(t) = 0}
$I_n(t, x)\,dx$ = P{C(t) = 0, X(t) = n, x ≤ ξ(t) < x + dx}, n ≥ 1
$P_{i,n}(t, x)\,dx$ = P{C(t) = i, X(t) = n, x ≤ ξ(t) < x + dx}, n ≥ 0 ; i = 1, 2, ...,M
$R_{i,n}(t, x)\,dx$ = P{C(t) = M+i, X(t) = n, x ≤ ξ(t) < x + dx}, n ≥ 0 ; i = 1, 2, ...,M
$V_{i,n}(t, x)\,dx$ = P{C(t) = 2M+i, X(t) = n, x ≤ ξ(t) < x + dx}, n ≥ 0 ; i = 1, 2, ...,M

Let I_0, $I_n(x)$, $P_{i,n}(x)$, $R_{i,n}(x)$ and $V_{i,n}(x)$ be the steady state probabilities of $I_0(t)$, $I_n(t,x)$, $P_{i,n}(t,x)$, $R_{i,n}(t,x)$ and $V_{i,n}(t,x)$ (i = 1, 2, . . . , M) respectively.

Then the steady state equations are

$$\lambda^+ I_0 = \sum_{i=1}^{M} \left[\bar{\tau} q_i \int_0^\infty P_{i,0}(x)\, \mu_i(x)\, dx + \int_0^\infty R_{i,0}(x)\, \beta_i(x)\, dx + \int_0^\infty V_{i,0}(x)\, \gamma_i(x)\, dx \right] \quad (4.1)$$

$$\frac{d\, I_n(x)}{dx} = -(\lambda_1^+ + \eta(x)) I_n(x)\ , \qquad n \geq 1 \quad (4.2)$$

$$\frac{dP_{i,n}(x)}{dx} = -(\lambda_2^+ + \lambda^- + \mu_i(x)) P_{i,n}(x) + \lambda^+ (1 - \delta_{0n}) \sum_{k=1}^{n} a_{2,k} P_{i,n-k}(x), \quad n \geq 0\ ;\ i = 1,2,....,M \quad (4.3)$$

$$\frac{dR_{i,n}(x)}{dx} = -(\lambda_3^+ + \beta_i(x)) R_{i,n}(x) + \lambda^+ (1 - \delta_{0n}) \sum_{k=1}^{n} a_{3,k} R_{i,n-k}(x), \quad n \geq 0\ ;\ i = 1,2,....,M \quad (4.4)$$

$$\frac{dV_{i,n}(x)}{dx} = -(\lambda_4^+ + \gamma_i(x))V_{i,n}(x) + \lambda^+(1-\delta_{0n})\sum_{k=1}^{n} a_{4,k}V_{i,n-k}(x), \quad n \geq 0\ ;\ i = 1,2,....,M \qquad (4.5)$$

with boundary conditions

$$I_n(0) = \bar{\tau}\sum_{i=1}^{M}\left[\left(q_i\int_0^\infty P_{i,n}(x)\,\mu_i(x)\,dx + \delta_i\int_0^\infty P_{i,n-1}(x)\,\mu_i(x)\,dx\right)\right]$$

$$+\sum_{i=1}^{M}\left[\int_0^\infty R_{i,n}(x)\,\beta_i(x)\,dx + \int_0^\infty V_{i,n}(x)\,\gamma_i(x)\,dx\right], \quad n \geq 1 \qquad (4.6)$$

$$P_{1,0}(0) = \lambda^+ a_{1,1}I_0 + \int_0^\infty I_1(x)\,\eta(x)\,dx \qquad (4.7)$$

$$P_{1,n}(0) = \lambda^+ a_{1,n+1}I_0 + \lambda^+\sum_{k=1}^{n} a_{1,k}\int_0^\infty I_{n-k+1}(x)dx + \int_0^\infty I_{n+1}(x)\,\eta(x)\,dx, \quad n \geq 1 \qquad (4.8)$$

$$P_{i,n}(0) = \theta_{i-1}\int_0^\infty P_{i-1,n}(x)\,\mu_{i-1}(x)\,dx, \qquad n \geq 0;\ i = 2,3,.....,M \qquad (4.9)$$

$$R_{i,n}(0) = \lambda^-\int_0^\infty P_{i,n}(x)\ dx, \qquad n \geq 0;\ i = 2,3,.....,M \qquad (4.10)$$

$$V_{i,n}(0) = \tau\sum_{i=1}^{M}\left[\left(q_i\int_0^\infty P_{i,n}(x)\,\mu_i(x)\,dx + \delta_i\int_0^\infty P_{i,n-1}(x)\,\mu_i(x)\,dx\right)\right] \qquad n \geq 0;\ i = 2,3,.....,M \qquad (4.11)$$

System Size Distribution at Random Epoch

Define the joint probability generating functions :

$$I(z,x) = \sum_{n=1}^{\infty} I_n(x)\,z^n \quad ; \quad P_i(z,x) = \sum_{n=0}^{\infty} P_{i,n}(x)\,z^n$$

$$R_i(z,x) = \sum_{n=0}^{\infty} R_{i,n}(x)\,z^n \quad ; \quad V_i(z,x) = \sum_{n=0}^{\infty} V_{i,n}(x)\,z^n, \quad i = 1,2,....., M$$

Using the definitions of I(z, x) , P_i(z, x) , R_i(z, x) and V_i(z, x), i = 1, 2, … M, we obtain the following equations from the equations (4.2) to (4.11)

$$\left(\frac{\partial}{\partial x} + \lambda_1^+ + \eta(x)\right)I(z,x) = 0 \qquad (4.12)$$

$$\left(\frac{\partial}{\partial x} + \lambda^- + \lambda_2^+(1-a_2(z)) + \mu_i(x)\right)P_i(z,x) = 0\ , \qquad i = 1,2,....,M \qquad (4.13)$$

$$\left(\frac{\partial}{\partial x} + \lambda_3^+(1-a_3(z)) + \beta_i(x)\right)R_i(z,x) = 0\ , \qquad i = 1,2,....,M \qquad (4.14)$$

$$\left(\frac{\partial}{\partial x} + \lambda_4^+(1-a_4(z)) + \gamma_i(x)\right)V_i(z,x) = 0\ , \qquad i = 1,2,....,M \qquad (4.15)$$

$$I(z,0) = \bar{\tau}\sum_{i=1}^{M}(q_i + z\delta_i)\int_0^\infty P_i(z,x)\,\mu_i(x)\,dx + \sum_{i=1}^{M}\left[\int_0^\infty R_i(z,x)\beta_i(x)dx + \int_0^\infty V_i(z,x)\gamma_i(x)dx\right] - \lambda_1^+ I_0 \qquad (4.16)$$

$$zP_1(z,0) = \lambda^+ I_0 a_1(z) + \lambda^+ a_1(z)\int_0^\infty I(z,x)dx + \int_0^\infty I(z,x)\,\eta(x)\,dx \qquad (4.17)$$

$$P_i(z,0) = \theta_{i-1}\int_0^\infty \mathrm{P}_{i-1,n}(z,x)\mu_{i-1}(x)dx\,, \qquad n \ge 0, \qquad i = 2,3,\ldots\ldots, M \tag{4.18}$$

$$R_i(z,0) = \lambda^-\int_0^\infty \mathrm{P}_{i,n}(z,x)\,dx\,, \qquad n \ge 0, \qquad i = 2,3,\ldots\ldots, M \tag{4.19}$$

$$V_i(z,0) = \tau(q_i + \delta_i z)\int_0^\infty \mathrm{P}_{i,n}(z,x)\mu_i(x)dx\,, \qquad n \ge 0, \qquad i = 2,3,\ldots\ldots, M \tag{4.20}$$

Solving the partial differential equations (4.12) - (4.15) we get

$$I(z,x) = I(z,0)e^{-\lambda_1^+ x}[1 - A(x)] \tag{4.21}$$

$$P_i(z,x) = P_i(z,0)e^{-(\lambda^+ + \lambda^- - \lambda^+ a_2(z))x}[1 - B_i(x)] \tag{4.22}$$

$$R_i(z,x) = R_i(z,0)e^{-(\lambda^+ - \lambda^+ a_3(z))x}[1 - R_i(x)] \tag{4.23}$$

$$V_i(z,x) = V_i(z,0)e^{-(\lambda^+ - \lambda^+ a_4(z))x}[1 - V_i(x)] \tag{4.24}$$

Using the expressions in equations (4.21) - (4.24) , equations (4.16) – (4.20) yield

$$I(z,0) = \bar{\tau}\sum_{i=1}^{M}[\mathrm{q_i} + \delta_i\, \mathrm{z}]\,\mathrm{P_i}(\mathrm{z},0)\,\mathrm{B_i^*}(\mathrm{g(z)})$$

$$+\sum_{\mathrm{i=1}}^{\mathrm{M}}\Big[\mathrm{R_i}(\mathrm{z},0)\,\mathrm{R_i^*}(\lambda^+(1-\mathrm{a_3(z)}) + \mathrm{V_i}(\mathrm{z},0)\,\mathrm{V_i^*}(\lambda^+(1-\mathrm{a_4(z)})\Big] - \lambda_1^+\,\mathrm{I_0} \tag{4.25}$$

$$zP_1(z,0) = \lambda^+\big(a_1(z) - a_{1,0}\big)I_0 + [A^*(\lambda_1^+) + \frac{\lambda^+}{\lambda^-}\big(a_1(z) - a_{1,0}\big)(1 - A^*(\lambda_1^+))]I(0,z) \tag{4.26}$$

$$P_i(z,0) = Q_{i-1}\Lambda_{i-1}^*(g(z))P_1(z,0)\,, \qquad i = 2,3,\ldots\ldots, M \tag{4.27}$$

$$R_i(z,0) = \lambda^- Q_{i-1}\Lambda_{i-1}^*(g(z))\overline{B_i^*}(g(z))/g(z)\;P_1(z,0)\,, \qquad i = 2,3,\ldots\ldots, M \tag{4.28}$$

$$V_i(z,0) = \tau(q_i + \delta_i z)Q_{i-1}\Lambda_i^*(g(z))P_1(z,0)\,, \qquad i = 2,3,\ldots\ldots, M \tag{4.29}$$

where $\Lambda_i^* = B_1^* B_2^* \ldots\ldots B_i^*$ *with* $\Lambda_0^* = 1$, $Q_{i-1} = \theta_1\,\theta_2\,\theta_3\ldots\ldots\,\theta_{i-1}$ with $Q_0 = 1$

and $g(z) = \lambda^+ + \lambda^- - \lambda^+ a_2(z)$ *and* $\overline{B_i^*}(g(z)) = 1 - B_i^*(g(z))$

Substituting the expressions in equations (4.25), (4.27), (4.28) and (4.29), the equation (4.26) becomes

$$P_1(z,0) = I_0[\lambda^+\big(a_1(z) - a_{1,0}\big) - \lambda_1^+ T_1(z)]/D(z) \tag{4.30}$$

where $D(z) = z - T_1(z)T_2(z)$

$$T_1(z) = A^*(\lambda_1^+) + \frac{\lambda^+}{\lambda^-}\big(a_1(z) - a_{1,0}\big)(1 - A^*(\lambda_1^+))$$

$$T_2(z) = \bar{\tau}\sum_{i=1}^{M}[\mathrm{q_i} + \delta_i\,\mathrm{z}]\,\mathrm{Q_{i-1}}\Lambda_i^*(\mathrm{g(z)}) + \lambda^-\sum_{\mathrm{i=1}}^{\mathrm{M}}\mathrm{Q_{i-1}}\Lambda_{\mathrm{i-1}}^*(\mathrm{g(z)})\,\overline{\mathrm{B_i^*}}(\mathrm{g(z)})\big/\mathrm{g(z)}\,\mathrm{R_i^*}(\lambda^+(1-\mathrm{a_3(z)})$$

$$+\tau\sum_{\mathrm{i=1}}^{\mathrm{M}}[\mathrm{q_i} + \delta_i]Q_{i-1}\Lambda_i^*(g(z))\mathrm{V_i^*}(\lambda^+(1-\mathrm{a_4(z)})$$

Using equation (4.30), equations (4.27)-(4.29) yield

$$P_i(z,0) = I_0 Q_{i-1}\Lambda_{i-1}^*(g(z))[\lambda^+\big(a_1(z) - a_{1,0}\big) - \lambda_1^+ T_1(z)]/D(z) \tag{4.31}$$

$$R_i(z,0) = I_0\lambda^- Q_{i-1}\Lambda_{i-1}^*(g(z))\,\overline{\mathrm{B_i^*}}(\mathrm{g(z)})\big/\mathrm{g(z)}[\lambda^+\big(a_1(z) - a_{1,0}\big) - \lambda_1^+ T_1(z)]/D(z) \tag{4.32}$$

$$V_i(z,0) = I_0\tau[q_i + \delta_i z]Q_{i-1}\Lambda_i^*(g(z))[\lambda^+\big(a_1(z) - a_{1,0}\big) - \lambda_1^+ T_1(z)]/D(z) \tag{4.33}$$

Consequently I(z, 0) in the equation (4.25) becomes

$$I(z,0) = I_0\Big[T_2(z)[\lambda^+\big(a_1(z) - a_{1,0}\big) - \lambda_1^+ T_1(z)]\Big] - \lambda_1^+[z - T_1(z)T_2(z)]/D(z) \tag{4.34}$$

The probability generating function of the orbit size when the server is idle is given by

$$I(z) = \int_0^\infty \mathrm{I(z, x)\, dx}$$

$$= I_0\left(1 - A^*(\lambda_1^+)\right)\left[T_2(z)[\lambda^+\left(a_1(z) - a_{1,0}\right) - \lambda_1^+ T_1(z)]\right] - \lambda_1^+[z - T_1(z)T_2(z)] / \lambda_1^+ D(z) \quad (4.35)$$

The probability generating function of the orbit size when the server is busy in providing ith (i = 1, 2, ..., M) stage service is

$$P_i(z) = \int_0^\infty \mathrm{P_i(z, x)\, dx}$$

$$= I_0 Q_{i-1}\Lambda_{i-1}^*(g(z))\overline{\mathrm{B}_i^*}(\mathrm{g(z)})[\lambda^+\left(a_1(z) - a_{1,0}\right) - \lambda_1^+ T_1(z)] / g(z)D(z) \quad (4.36)$$

The probability generating function of the orbit size when the server is repair at ith (i = 1, 2, ..., M) stage service is

$$R_i(z) = \int_0^\infty \mathrm{R_i(z, x)\, dx}$$

$$= \frac{I_0 \lambda^- Q_{i-1}\Lambda_{i-1}^*(g(z))\overline{\mathrm{B}_i^*}(\mathrm{g(z)}) / \mathrm{g(z)}[\lambda^+\left(a_1(z) - a_{1,0}\right) - \lambda_1^+ T_1(z)][1 - R_i^*(\lambda^+(1 - a_3(z)))]}{\lambda^+(1 - a_3(z))g(z)D(z)} \quad (4.37)$$

The probability generating function of the orbit size when the server takes vacation after ith (i = 1, 2, ..., M) stage service is

$$V_i(z) = \int_0^\infty \mathrm{V_i(z, x)\, dx}$$

$$= \frac{I_0 \tau[q_i + \delta_i z]Q_{i-1}\Lambda_i^*(g(z))[\lambda^+\left(a_1(z) - a_{1,0}\right) - \lambda_1^+ T_1(z)][1 - V_i^*(\lambda^+(1 - a_4(z)))]}{\lambda^+(1 - a_4(z))D(z)} \quad (4.38)$$

Using the normalizing condition

$$I_0 + \lim_{z\to 1} I(z) + \lim_{z\to 1}\sum_{i=1}^{M} \mathrm{P_i}(z) + \lim_{z\to 1}\sum_{i=1}^{M} \mathrm{R_i}(z) + \lim_{z\to 1}\sum_{i=1}^{M} \mathrm{V_i}(z) = 1$$

$$we \quad obtain \quad I_0 = [1 - T_1'(1) - T_2'(1)] / [A^*(\lambda)\Big\{(1 - T_1'(1) - T_2'(1)) + \lambda^+\chi_1 m_1[\left(1 - A^*(\lambda_1^+)\right) / \lambda_1^+$$

$$+ \sum_{i=1}^{M} Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-) / \lambda^- + \sum_{i=1}^{M} Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)\beta_{i,1} + \tau\sum_{i=1}^{M}(q_i + \delta_i)Q_{i-1}\Lambda_i^*(\lambda^-)\gamma_{i,1}] \Big\}] \quad (4.39)$$

$$where \quad T_1'(1) = (\lambda^+ / \lambda_1^+)\chi_1 m_1\left(1 - A^*(\lambda_1^+)\right)$$

$$and \quad T_2'(1) = \sum_{i=1}^{M}\delta_i Q_{i-1}\Lambda_i^*(\lambda^-) + \sum_{i=1}^{M}(q_i + \delta_i)Q_{i-1}M_{i,1} + \sum_{i=1}^{M} Q_{i-1}M_{i-1,1}\overline{B}_i^*(\lambda^-)$$

$$+ \sum_{i=1}^{M} Q_{i-1}M_{i,1}\Lambda_{i-1}^*(\lambda^-) + \lambda^+\chi_2 m_1\sum_{i=1}^{M} Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B}_i^*(\lambda^-) / \lambda^-$$

$$+ \chi_3 m_1\sum_{i=1}^{M} Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B}_i^*(\lambda^-)\beta_{i,1} + \tau\chi_4 m_1\sum_{i=1}^{M}(q_i + \delta_i)Q_{i-1}\Lambda_i^*(\lambda^-)\gamma_{i,1}$$

$$T_1''(1) = (\lambda^+ / \lambda_1^+)\chi_1 m_1 \chi_1^2 m_2\left(1 - A^*(\lambda_1^+)\right)$$

$$T_2''(1) = 2\sum_{i=1}^{M}\delta_i Q_{i-1}\Lambda_i^*(\lambda^-) + \sum_{i=1}^{M}(q_i + \delta_i)Q_{i-1}M_{2,i} - 2\sum_{i=1}^{M} Q_{i-1}M_{1,i-1}\mu_{i,1} + 2\lambda^+\chi_2 m_1\sum_{i=1}^{M} Q_{i-1}M_{1,i-1}\overline{B}_i^*(\lambda^-) / \lambda^-$$

$$-\sum_{i=1}^{M} Q_{i-1}\Lambda_{i-1}^{*}(\lambda^{-})\mu_{i,2}/\lambda^{-}+\lambda^{+}\chi_2^2 m_2\sum_{i=1}^{M} Q_{i-1}\Lambda_{i-1}^{*}(\lambda^{-})\overline{B}_i^{*}(\lambda^{-})/(\lambda^{-})^3+\lambda^{+}\chi_2 m_1\sum_{i=1}^{M} Q_{i-1}\Lambda_{i-1}^{*}(\lambda^{-})\mu_{i,1}/(\lambda^{-})^3$$

$$+(\lambda^{+})^3\chi_2 m_1\chi_2^2 m_2\sum_{i=1}^{M} Q_{i-1}\Lambda_{i-1}^{*}(\lambda^{-})\overline{B}_i^{*}(\lambda^{-})/(\lambda^{-})^4+\sum_{i=1}^{M} Q_{i-1}M_{2,i-1}\overline{B}_i^{*}(\lambda^{-})/\lambda^{-}+2\sum_{i=1}^{M} Q_{i-1}M_{1,i-1}\overline{B}_i^{*}(\lambda^{-})/(\lambda^{-})\beta_{i,1}$$

$$-2\sum_{i=1}^{M} Q_{i-1}\Lambda_{i-1}^{*}(\lambda^{-})M_{1,i}\beta_{i,1}+2\lambda^{+}\chi_2 m_1\sum_{i=1}^{M} Q_{i-1}\Lambda_{i-1}^{*}(\lambda^{-})\overline{B}_i^{*}(\lambda^{-})/(\lambda^{-})\beta_{i,1}+2\sum_{i=1}^{M} Q_{i-1}\Lambda_{i-1}^{*}\overline{B}_i^{*}(\lambda^{-})\beta_{i,2}$$

$$+2\tau\sum_{i=1}^{M}(q_i+\delta_i)Q_{i-1}M_{1,i}\gamma_{i,1}+\tau\sum_{i=1}^{M}\delta_i Q_{i-1}\Lambda_i^{*}(\lambda^{-})\gamma_{i,1}+\tau\sum_{i=1}^{M}(q_i+\delta_i)Q_{i-1}\Lambda_i^{*}(\lambda^{-})\gamma_{i,2}$$

The probability generating function of the orbit size is

$$P_q(z)=I_0+I(z)+\sum_{i=1}^{M}\mathrm{P_i}(z)+\sum_{i=1}^{M}\mathrm{R_i}(z)+\sum_{i=1}^{M}\mathrm{V_i}(z)$$

$$=\mathrm{I_0}\Big\{\ \mathrm{A}^{*}(\lambda)[\mathrm{z}-\mathrm{T_1}(z)T_2(z)]+[\lambda^{+}\big(a_1(z)-a_{1,0}\big)-\lambda_1^{+}T_1(z)][\big(1-A^{*}(\lambda_1^{+}\big)T_2(z)/\lambda_1^{+}$$

$$+\sum_{i=1}^{M} Q_{i-1}\Lambda_{i-1}^{*}(g(z))\overline{\mathrm{B}_i^{*}}(\mathrm{g(z)})/g(z)+\lambda^{-}Q_{i-1}\Lambda_{i-1}^{*}(g(z))\overline{\mathrm{B}_i^{*}}(\mathrm{g(z)})\big/\mathrm{g(z)}[1-R_i^{*}(\lambda^{+}(1-a_3(z)))]$$

$$/(\lambda^{+}(1-a_3(z)))+\tau\sum_{i=1}^{M}[q_i+\delta_i z]Q_{i-1}\Lambda_i^{*}(g(z))[1-V_i^{*}(\lambda^{+}(1-a_4(z)))]/(\lambda^{+}(1-a_4(z)))]\ \Big\}/D(z)$$

(4.40)

The probability generating function of the system size is

$$P_s(z)=I_0+I(z)+z\sum_{i=1}^{M}\mathrm{P_i}(z)+\sum_{i=1}^{M}\mathrm{R_i}(z)+\sum_{i=1}^{M}\mathrm{V_i}(z)$$

$$=\mathrm{I_0}\Big\{\ \mathrm{A}^{*}(\lambda)[\mathrm{z}-\mathrm{T_1}(z)T_2(z)]+[\lambda^{+}\big(a_1(z)-a_{1,0}\big)-\lambda_1^{+}T_1(z)][\big(1-A^{*}(\lambda_1^{+}\big)T_2(z)/\lambda_1^{+}$$

$$+z\sum_{i=1}^{M} Q_{i-1}\Lambda_{i-1}^{*}(g(z))\overline{\mathrm{B}_i^{*}}(\mathrm{g(z)})/g(z)+\lambda^{-}Q_{i-1}\Lambda_{i-1}^{*}(g(z))\overline{\mathrm{B}_i^{*}}(\mathrm{g(z)})\big/\mathrm{g(z)}[1-R_i^{*}(\lambda^{+}(1-a_3(z)))]$$

$$/(\lambda^{+}(1-a_3(z)))+\tau\sum_{i=1}^{M}[q_i+\delta_i z]Q_{i-1}\Lambda_i^{*}(g(z))[1-V_i^{*}(\lambda^{+}(1-a_4(z)))]/(\lambda^{+}(1-a_4(z)))]\ \Big\}/D(z)$$

(4.41)

Performance Measures

- The probability that the server is idle in the non-empty system is given by

$$I=\lim_{z\to 1}I(z)$$

$$=\frac{I_0\big(1-A^{*}(\lambda_1^{+})\big)\Big\{\lambda^{+}\chi_1 m_1 A^{*}(\lambda_1^{+})-\lambda_1^{+}[1-T_1^{'}(1)-T_2^{'}(1)]\Big\}}{\lambda_1^{+}[1-T_1^{'}(1)-T_2^{'}(1)]} \qquad (4.42)$$

- The probability that the server is busy is

$$P=\lim_{z\to 1}\sum_{i=1}^{M}P_i(z)$$

$$=\frac{\lambda^{+}I_0\sum_{i=1}^{M} Q_{i-1}\Lambda_{i-1}^{*}(\lambda^{-})\chi_1 m_1 A^{*}(\lambda_1^{+})\overline{B_i^{*}}(\lambda^{-})}{\lambda^{-}[1-T_1^{'}(1)-T_2^{'}(1)]} \qquad (4.43)$$

- The probability that the server is under repair is

$$R = \lim_{z\to 1}\sum_{i=1}^{M} R_i(z)$$

$$= \frac{\lambda^+\lambda^- I_0 \sum_{i=1}^{M} Q_{i-1}\Lambda^*_{i-1}(\lambda^-)\chi_1 m_1 A^*(\lambda_1^+)\overline{B_i^*}(\lambda^-)\beta_{i,1}}{[1-T_1'(1)-T_2'(1)]} \qquad (4.44)$$

- The probability that the server is on vacation is

$$V = \lim_{z\to 1}\sum_{i=1}^{M} V_i(z)$$

$$= \frac{\tau\lambda^+ I_0 \sum_{i=1}^{M}(q_i+\delta_i)Q_{i-1}\Lambda^*_i(\lambda^-)\chi_1 m_1 A^*(\lambda_1^+)\gamma_{i,1}}{[1-T_1'(1)-T_2'(1)]} \qquad (4.45)$$

- Mean number of customers when the server is in idle state is

$$L_I = \lim_{z\to 1}\frac{d}{dz}I(z)$$

$$= \frac{\begin{aligned}&\left(1-A^*(\lambda_1^+)\right)/\lambda_1^+[1-T_1'(1)-T_2'(1)]\Big\{2T_2'(1)[\lambda^+\chi_1 m_1-\lambda_1^+T_1'(1)]+[\lambda^+\chi_1^2 m_2-\lambda_1^+T_2''(1)]\\ &-[\lambda^+\chi_1 m_1-\lambda_1^+T_1'(1)][1-T_1''(1)2T_1'(1)T_2'(1)-T_2''(1)]\Big\}\end{aligned}}{\begin{aligned}&2[1-T_1'(1)-T_2'(1)][A^*(\lambda)\Big\{(1-T_1'(1)-T_2'(1))+\lambda^+\chi_1 m_1[\left(1-A^*(\lambda_1^+)\right)/\lambda_1^+\\ &+\sum_{i=1}^{M}Q_{i-1}\Lambda^*_{i-1}(\lambda^-)\overline{B_i^*}(\lambda^-)/\lambda^- + \sum_{i=1}^{M}Q_{i-1}\Lambda^*_{i-1}(\lambda^-)\overline{B_i^*}(\lambda^-)\beta_{i,1}+\tau\sum_{i=1}^{M}(q_i+\delta_i)Q_{i-1}\Lambda^*_i(\lambda^-)\gamma_{i,1}]\Big\}\end{aligned}} \qquad (4.46)$$

- Mean number of customers when the server is busy is

$$L_P = \frac{\begin{aligned}&[1-T_1'(1)-T_2'][2\sum_{i=1}^{M}Q_{i-1}M_{1,i-1}(\lambda^+\chi_1 m_1-\lambda_1^+T_1'(1))\overline{B_i^*}(\lambda^-)/\lambda^- + \sum_{i=1}^{M}Q_{i-1}\Lambda^*_{i-1}(\lambda^-)\overline{B_i^*}(\lambda^-)/\lambda^-\\ &(\lambda^+\chi_1^2 m_2-\lambda_1^+T_1''(1))-2\sum_{i=1}^{M}Q_{i-1}M_{1,i}(\lambda^+\chi_1 m_1-\lambda_1^+T_1'(1))\Lambda^*_{i-1}(\lambda^-)/\lambda^- - 2\sum_{i=1}^{M}Q_{i-1}\Lambda^*_{i-1}(\lambda^-)\lambda^+\chi_2 m_1\\ &\overline{B_i^*}(\lambda^-)/(\lambda^-)^2]-[1-T_1''(1)-T_1'(1)T_2'(1)-T_2''(1)][\lambda^+\chi_1 m_1-\lambda_1^+T_1'(1)]\sum_{i=1}^{M}Q_{i-1}\Lambda^*_{i-1}(\lambda^-)\overline{B_i^*}(\lambda^-)/\lambda^-\end{aligned}}{\begin{aligned}&2[1-T_1'(1)-T_2'(1)]A^*(\lambda_1^+)\Big\{[1-T_1'(1)-T_2'(1)]+\lambda^+\chi_1 m_1[(1-A^*(\lambda_1^+))/\lambda_1^+\\ &+\sum_{i=1}^{M}Q_{i-1}\Lambda^*_{i-1}(\lambda^-)\overline{B_i^*}(\lambda^-)/\lambda^- + \sum_{i=1}^{M}Q_{i-1}\Lambda^*_{i-1}(\lambda^-)\overline{B_i^*}(\lambda^-)\beta_{i,1}+\tau\sum_{i=1}^{M}(q_i+\delta_i)Q_{i-1}\Lambda^*_i(\lambda^-)\gamma_{i,1}\end{aligned}} \qquad (4.47)$$

- Mean number of customers when the server is under repair is

$$L_R = \frac{\begin{array}{l}[1-T_1'(1)-T_2'][2\sum_{i=1}^{M}Q_{i-1}M_{1,i-1}(\lambda^+\chi_1 m_1-\lambda_1^+T_1'(1))\overline{B_i^*}(\lambda^-)\beta_{i,1}-\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)M_{1,i}\beta_{i,1}\\ +\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\lambda^+\chi_2 m_1\overline{B_i^*}(\lambda^-)\beta_{i,1}+\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)[\lambda^+\chi_3^2 m_2\beta_{i,1}-\lambda^+\chi_3^2 m_2\beta_{i,2}]/(\chi_3 m_1)+\\ (\lambda^+\chi_1 m_2-\lambda_1^+T_1''(1))\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)\beta_{i,1}]-[1-T_1''(1)-T_1'(1)T_2'(1)-T_2''(1)][\lambda^+\chi_1 m_1-\lambda_1^+T_1'(1)]\\ (\lambda^+\chi_1^2 m_2-\lambda_1^+T_1''(1))\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)\beta_{i,1}]\end{array}}{\begin{array}{l}2[1-T_1'(1)-T_2'(1)]A^*(\lambda_1^+)\{[1-T_1'(1)-T_2'(1)]+\lambda^+\chi_1 m_1[(1-A^*(\lambda_1^+))/\lambda_1^+\\ +\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)/\lambda^-+\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)\beta_{i,1}+\tau\sum_{i=1}^{M}(q_i+\delta_i)Q_{i-1}\Lambda_i^*(\lambda^-)\gamma_{i,1}\end{array}} \quad (4.48)$$

- Mean number of customers when the server is on vacation is

$$L_V = \frac{\begin{array}{l}\tau[1-T_1'(1)-T_2'][2(\lambda^+\chi_1 m_1-\lambda_1^+T_1'(1))\sum_{i=1}^{M}\delta_i Q_{i-1}\Lambda_i^*(\lambda^-)\gamma_{i,1}-\sum_{i=1}^{M}(q_i+\delta_i)Q_{i-1}\Lambda_i^*(\lambda^-)M_{1,i}\gamma_{i,1}\\ +\sum_{i=1}^{M}(q_i+\delta_i)Q_{i-1}\Lambda_i^*(\lambda^-)[\lambda^+\chi_4^2 m_2\gamma_{i,1}-\lambda^+\chi_4^2 m_2\gamma_{i,2}]/(\chi_4 m_1)+(\lambda^+\chi_1^2 m_2-\lambda_1^+T_1''(1))\sum_{i=1}^{M}(q_i+\delta_i)Q_{i-1}\Lambda_i^*(\lambda^-)\gamma_{i,1}]\\ -[1-T_1''(1)-T_1'(1)T_2'(1)-T_2''(1)][\lambda^+\chi_1 m_1-\lambda_1^+T_1'(1)](\lambda^+\chi_1^2 m_2-\lambda_1^+T_1''(1))\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)\gamma_{i,1}]\end{array}}{\begin{array}{l}2[1-T_1'(1)-T_2'(1)]A^*(\lambda_1^+)\{[1-T_1'(1)-T_2'(1)]+\lambda^+\chi_1 m_1[(1-A^*(\lambda_1^+))/\lambda_1^+\\ +\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)/\lambda^-+\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)\beta_{i,1}+\tau\sum_{i=1}^{M}(q_i+\delta_i)Q_{i-1}\Lambda_i^*(\lambda^-)\gamma_{i,1}\end{array}} \quad (4.49)$$

- Mean number of customers in the orbit is given by

$$L_q = \lim_{z\to 1}\frac{d\,\mathrm{P_q}(z)}{dz}$$

$$= \frac{\begin{array}{l}[\lambda^+\chi_1 m_2-\lambda_1^+T_1''(1)]T_3(1)+2[\lambda^+\chi_1^2 m_2-\lambda_1^+T_1''(1)]T_3'(1)\\ -[1-T_1''(1)-T_1'(1)T_2'(1)-T_2''(1)][\lambda^+\chi_1 m_1-\lambda_1^+T_1'(1)]T_3(1)\end{array}}{\begin{array}{l}2[1-T_1'(1)-T_2'(1)]A^*(\lambda_1^+)\{[1-T_1'(1)-T_2'(1)]+\lambda^+\chi_1 m_1[(1-A^*(\lambda_1^+))/\lambda_1^+\\ +\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)/\lambda^-+\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)\beta_{i,1}+\tau\sum_{i=1}^{M}(q_i+\delta_i)Q_{i-1}\Lambda_i^*(\lambda^-)\gamma_{i,1}\end{array}}$$

$$L_q = L_I + L_P + L_W + L_V \quad (4.50)$$

where $T_3(1) = (T_2(1)/\lambda_1^+)(1-A^*(\lambda_1^+))+\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)/\lambda^- + \sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)\beta_{i,1}$

$$+\tau\sum_{i=1}^{M}(q_i+\delta_i)Q_{i-1}\Lambda_i^*(\lambda^-)\gamma_{i,1}$$

$$T_3'(1) = (T_2'(1)/\lambda_1^+)(1-A^*(\lambda_1^+))+\sum_{i=1}^{M}Q_{i-1}M_{1,i-1}\overline{B_i^*}(\lambda^-)/\lambda^- - \lambda^+/\lambda^-\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\chi_2 m_1\mu_{i,1}$$

$$+\lambda^{+}\chi_2 m_1\sum_{i=1}^{M}Q_{i-1}M_{1,i-1}\overline{B_i^*}(\lambda^-)/(\lambda^-)^2+\sum_{i=1}^{M}Q_{i-1}B_i^*(\lambda^-)M_{1,i-1}\beta_{i,1}+\lambda^{+}\chi_2 m_1\sum_{i=1}^{M}Q_{i-1}\Lambda_i^*(\lambda^-)$$

$$+\lambda^{+}\chi_2 m_1\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)/(\lambda^-)\beta_{i,1}-\lambda^{+}\chi_3^2 m_2\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)\beta_{i,2}$$

$$+\lambda^{+}\chi_3^2 m_2\sum_{i=1}^{M}Q_{i-1}\Lambda_{i-1}^*(\lambda^-)\overline{B_i^*}(\lambda^-)\beta_{i,1}+\tau\sum_{i=1}^{M}\delta_i Q_{i-1}\Lambda_i^*(\lambda^-)\gamma_{i,1}+\tau\sum_{i=1}^{M}(q_i+\delta_i)Q_{i-1}M_{1,i}\gamma_{i,1}$$

$$+\tau\lambda^{+}\chi_4^2 m_2\sum_{i=1}^{M}(q_i+\delta_i)Q_{i-1}\Lambda_i^*(\lambda^-)\gamma_{i,2}+\tau\lambda^{+}\chi_4^2 m_2\sum_{i=1}^{M}(q_i+\delta_i)Q_{i-1}\Lambda_i^*(\lambda^-)\gamma_{i,1}$$

- Mean number of customers in the system is

$$L_s=\lim_{z\to 1}\frac{dP_s(z)}{dz}$$
$$= L_q + P \qquad (4.51)$$

System Size Distribution at a Departure Epoch

Let Φ_j denote the probability that there are j units in the system at a departure epoch. Then

$$\Phi_j = K_0\sum_{i=1}^{M} q_i\int_0^{\infty} P_{i,j}(x)\,\mu_i(x)\,dx, \qquad j\ge 0 \qquad (4.52)$$

where K_0 is the normalizing constant.

The probability generating function $\Phi(z)$ of $\{\Phi_j, j\ge 0\}$ is given by

$$\Phi(z)=K_0\sum_{i=1}^{M} q_i\int_0^{\infty} P_i(z,x)\,\mu_i(x)\,dx$$

$$=K_0 I_0[\lambda^{+}(a_1(z)-a_{1,0})-\lambda_1^{+}T_1(z)]\sum_{i=1}^{M} Q_{i-1}\Lambda_i^*(g(z))/D(z) \qquad (4.53)$$

Now the constant K_0 can be determined by using the normalizing condition $\Phi(1) = 1$, as

$$K_0 = \frac{[1-T_1'(1)-T_2'(1)]}{I_0[\lambda^{+}\chi_1 m_1-\lambda_1^{+}T_1'(1)]\sum_{i=1}^{M} Q_{i-1}\Lambda_i^*(\lambda^-)} \qquad (4.54)$$

Substituting K_0 in the equation (4.53), we obtain the probability generating function of the number of customers in the system at a departure epoch as

$$\Phi(z)=\frac{[1-T_1'(1)-T_2'(1)][\lambda^{+}(a_1(z)-a_{1,0})-\lambda_1^{+}T_1(z)]\sum_{i=1}^{M} Q_{i-1}\Lambda_i^*(g(z))}{[\lambda^{+}\chi_1 m_1-\lambda_1^{+}T_1'(1)]\sum_{i=1}^{M} Q_{i-1}\Lambda_i^*(\lambda^-)D(z)} \qquad (4.55)$$

The expected number of units in the system at a departure epoch is

$$L_d=\lim_{z\to 1}\frac{d}{dz}\Phi(z)$$

$$= \frac{[\lambda^{+}\chi_1 m_2 - \lambda_1^{+} T_1^{''}(1)]\sum_{i=1}^{M} Q_{i-1}\Lambda_i^{*}(\lambda^{-}) + 2[\lambda^{+}\chi_1 m_1 - \lambda_1^{+} T_1^{''}(1)]\sum_{i=1}^{M} Q_{i-1} M_{1,i}}{2[\lambda^{+}\chi_1 m_1 - \lambda_1^{+} T_1'(1)]\sum_{i=1}^{M} Q_{i-1}\Lambda_i^{*}(\lambda^{-})}$$

$$+ \frac{[1 - T_1^{''}(1) - 2T_1^{'}(1)T_2^{'}(1) - T_2^{''}(1)]}{2[1 - T_1^{'}(1) - T_2^{'}(1)]} \qquad (4.56)$$

Conclusion

In this paper, state dependent bulk arrival multi-stage retrial G-queue with feedback and vacation is considered. The steady state analysis with probability generating functions for this model has been derived. Some performance measures are obtained.

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The *b*-Coloring of Corona graph of Path graph with Origami graph

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ABSTRACT: *A b-coloring of a graph G is a proper coloring of the vertices of G such that there exists a vertex in each color class joined to at least one vertex in each other color class. In graph the b – chromatic number, indicated by φ(G), is the maximal integer k such that G may have a b – coloring with k colors. In this paper, we obtain the b-chromatic number for the Corona product of Path graph with Origami graph denoted by* $\varphi(O_{n_0}P_n)$, $\varphi(P_{n_0}O_n)$.

Keywords: *Graph coloring, b-coloring, corona product,Path graph, Origami graph.*

1.Introduction

Let $G = (V, E)$ be a simple and finite graph without loops. Definitions used here may be found in [1]. A suitable k coloring is a function $c : V(G) \to \{1, 2, \ldots, k\}$ such that $c(u) = c(v)$ for all uv belongs to edge of G. The chromatic number $\chi(G)$ is the lowest integer k for which G declare proper k-coloring. Given a k-coloring c, a vertex v is called a b-vertex of color i, if $c(v) = i$ and v has at least one neighbour in each of the other color classes c_j, $i = j$. Then proper coloring c of graph G is a b-coloring if every color class has a b-vertex. The b-chromatic number of a graph G, denoted by $\phi(G)$, is the largest integer k such that G may have a b-coloring by k colors. The concept of b-chromatic number $\phi(G)$ was first introduced by Irving and Manlove [2]. They also proved the upper bound of $\phi(G)$, is $\phi(G) \le m(G)$, where $m(G)$ is the largest integer m such that G has m vertices of degree atleast $m - 1$. Kratochvl, Tuza and Voigt [3] evaluated the asymptotic behavior for the b-chromatic number of random graphs and proved that it is NP-completeness of the problem to choose whether there is a dominating appropriate k-coloring even for connected bipartite graphs and $k = \Delta(G) + 1$. Kouider and Zaker [4] proposed some upper bounds for the b-chromatic number of several classes of graphs in function of other graph parameters. Several related concepts concerning *b* coloring of graphs have been studied. Corona graph product was introduced by Frucht and Harary [5] in1970.

Origami is a kind of art first invented from Japan. It is a feasible to fold up a lot of good-looking figures in origami. Most miraculously, many amazing pieces of origami are shaped from a particular part of paper, with no cuttings. Justlike constructions using straight edge and compass, constructions through paperfolding is both mathematically interesting and aesthetic, particularly in origami graph [7]. In this paper, we obtain the *b*-chromatic number for the Corona product of Path graph with Origami graph.

2.Prelimaries

Path graph or **linear graph** is a graph whose vertices can be listed in the order $v_1, v_2, \ldots, v_n$ such that the edges are $\{v_i, v_{i+1}\}$ where $i = 1, 2, \ldots, n - 1$. Equally, a path with at least two vertices is connected and has two terminal vertices (vertices that have degree 1), while all others (if any) have degree 2. Paths are fundamental concepts of graph theory, described in the introductory sections of most graph theory texts.[6]

Corona product of G and H is the graph $G \circ H$ obtained by taking onecopy of G, called the center graph ,$|V(G)|$ copies of H, called the outer graph, and making the ith vertex of G adjacent to every vertex of the ith copy of H ,where $1 \le i \le |V(G)|$. The corona product is not commutative. For example, $K_3 \circ P_2$, $P_2 \circ K_3$ since the number of vertices differs. Also the corona product is not associative. It is easy to prove that $\chi(G \circ H) \le \Delta(G \circ H)+2$ for all G and H for the corona product

3. Main Results

b-colouring of Coronagraph of Origami graph with Path graph.

Theorem 1: For any positive integer n ,n≥ 4 *the b*-chromatic number of the corona graph of origami graph with path graph is $\varphi(O_n \circ P_n) = n+4$ for n≥ 4.

Proof:

Let $V(O_n)=\{a_i, b_i, d_i : 1\leq i \leq n\}$

$V(P_n)=\{P_i : 1\leq i\leq n\}$

Let $V(O_n \circ P_n)=\{a_i:1\leq i\leq n\}\cup\{b_i:1\leq i\leq n\}\cup\{d_i: 1\leq i\leq n\}\cup\{P_i: 1\leq i\leq n\}$

Consider n≥4, assign the following n+4 colors as *b*-chromatic number for $O_n \circ P_n$,

- For P_i:1≤i≤ n assign the colour ci: 1≤i≤ n+4 by the proper colouring procedure.
- For a_i:1≤i≤ 4, assign the colour $c_{\lceil\frac{n}{2}\rceil+i-1}$ and for the remaining vertices repeat the colors in the same order
- For b_i: 1≤i≤ n assign the color ci: n≤i≤ n+4 by the proper coloring procedure.
- And for d_i:1≤i≤ n assign the color ci

An easy check shows that this coloring is a *b*-coloring.
The coloring procedure depends the degree of the origami graph which is n+3
Then we can assign maximum number of n+4 colours to get a *b*-coloring.
Hence the proof.

b-colouring of Coronagraph of Path graph with Origami graph.

Theorem 2: For any positive integer n ,n≥ 4 *the b*-chromatic number of the corona graph of Path graph with Origami graph is $\varphi(P_n \circ O_n)$=n+1 for n≤ 6.
=n, for n≥ 7

Proof:

Let $V(P_n)=\{P_i : 1\leq i\leq n\}$

$V(O_n)=\{a_i, b_i, d_i, : 1\leq i \leq n\}$

Let $V(P_n \circ O_n) = \{P_i : 1\leq i\leq n\} \cup \{a_i : 1\leq i\leq n\} \cup \{b_i: 1\leq i \leq n\} \cup \{d_i: 1\leq i \leq n\}$

We prove the results by the following two cases.

Case 1:For n≤ 6.

Assign the following n+1 colors as *b*-chromatic number for $(P_n \circ O_n)$

- For P_i:1≤i≤ n, assign the color c_i.
- For a_i:1≤i≤ n, assign the color c_i: 1≤i≤ n+1,where color class c_i is not equal to color class of path.
- For b_i: 1≤i≤ 2, assign the color c_{i+3}, and for remaining vertices assign the colors c_i: 1≤i≤ $n+1$, by the proper coloring procedure.
- And for d_i:1≤i≤ 2, assign the color c_{i+4} and for remaining vertices assign the colors c_i: 2 ≤i≤ $n+1$ by the proper coloring procedure.

For the graph $(P_n \circ O_n)$, n≤ 6, the *b*-coloring procedure depends the vertex set of (O_n). O_n has n+1 vertices of degree n, then we can assign n+1 color to get *b*-coloring.

Case 2:For n≥ 7

Assign the following n colors as *b*-chromatic number for $(P_n \circ O_n)$

- For p_i:1≤i≤ n, assign the color c_i.
- For a_i:1≤i≤ n , assign the color c_i: 1≤i≤ n, with the proper coloring procedure.
- For b_i: 1≤i≤ n assign the color c_i: 1≤i≤ n, with the proper coloring procedure
- And for d_i:1≤i≤ n, assign the color c_i: 1≤i≤ n, with the proper coloring procedure.

For the graph $P_n \circ O_n$, n≥ 7, the *b*-coloring procedure depends the vertex set of (P_n). That is $P_i : 1\leq i \leq n$. This vertex set has n vertices of degree 3n+1.
Hence we can assign n colors to get *b*-coloring.
Hence the proof

4.conclusion

The *b*-coloring play an important role in clustering, automatic reading system and distributed system. We have investigated *b*- chromatic number of Corona graph of Pathgraph with Origami graph. The investigation of similar results for different graphs as well in the context of various

graph coloring problems is an open area of research.

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ANALYSIS OF SOCIAL MEDIA USAGE IN DIFFERENT AGE GROUPS BY VIKOR METHOD

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ABSTRACT: *The VIKOR method was developed for multi-criteria optimization of complex system. It determines the compromise ranking list and the compromise solution obtained with the initial weights. This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. It introduces the multi-criteria ranking index based on the particular measure of closeness to the ideal solution. The aim of this paper is to extend the VIKOR method for analyzing the usage of social media in various age group people.*

Keywords: *Multi criteria decision making, Social Media, VIKOR Method.*

1 INTRODUCTION

Decision making is the method of finding the best choice from all feasible alternatives. Much of the decision making in the real world takes place in an environment in which constraints and the consequences of possible actions are not known accurately and hence fuzzy set theory can be used to deal with imprecision in decision making. In classical multi criteria decision making (MCDM) methods the ratings and the weights of the criteria are known accurately [6].

The problems of Multi-Criteria Decision Making (MCDM) are intensely applied in many domains, such as Social Sciences, Medical Sciences, and Economics etc. MCDM problems are declared as Multi- Criteria Decision Analysis (MCDA) or Multi-Attribute Decision Making (MADM). In spite of their diversity, the MCDM have shared characteristic multiple objectives and multiple-criteria which usually are in conflict with each other. The decision makers have to assess or rank these alternatives according to the weights of the criteria. In the last decades, the MCDM techniques have become the main branch of operations research [5].

Multi Criteria Decision Making has grown as a part of operations research, concerned with designing mathematical and computational tools for supporting the subjective evaluation of performance criteria by decision makers [4]. Multi-criteria optimization (MCO) is considered as the process of determining the best feasible solution according to established criteria which represent different effects. However, these criteria usually conflict with each other and there may be no solution satisfying all criteria simultaneously. Thus, the concept of Pareto optimality was introduced for a vector optimization problem. Pareto optimal solutions have the characteristic that, if one criterion is to be improved, at least one other criterion has to be made worse. In such cases, a system analyst can aid the decision making process by making a comprehensive analysis and by listing the important properties of the Pareto optimal (non inferior) solutions. However, in engineering and management practice there is a need to select a final solution to be implemented. An approach to determine a final solution as a compromise was introduced by Yu, and other distance-based techniques have also been developed.

Many papers have proposed analytical models as aids in conflict management situations. Among the numerous approaches available for conflict management, one of the most prevalent is multi criteria decision making. Multi criteria decision making (MCDM) may be considered as a complex and dynamic process including one managerial level and one engineering level. The managerial level defines the goals, and chooses the final ‘‘optimal’’ alternative.

The multi criteria nature of decisions is emphasized at this managerial level, at which public officials called ‘‘decision makers’’, have the power to accept or reject the solution proposed by the engineering level. These decision makers, who provide the preference structure, are ‘‘off line’’ from the optimization procedure done at the engineering level.

Very often, the preference structure is based on political rather than only technical criteria. In such cases, a system analyst can aid the decision making process by making a comprehensive analysis and by

listing the important properties of non inferior and/or compromise solutions. The engineering level of the MCDM process defines alternatives and point out the consequences of choosing any one of them from the standpoint of various criteria. This level also performs the multi criteria ranking of alternatives.

VIKOR method is a multi-criteria decision making (MCDM) or multi-criteria decision analysis method. It was originally developed by Serafim Opricovic to solve decision problems with conflicting and non-commensurable (different units) criteria assuming that compromise is acceptable for conflict resolution, the decision maker wants a solution that is the closest to the ideal, and the alternatives are evaluated according to all established criteria. VIKOR ranks alternatives and determines the solution named compromise that is the closest to the ideal.

The VIKOR method includes a multi criteria optimization of complex systems that focuses on ranking and selecting from a set of alternatives among conflicting criteria [2]. It determines the compromise ranking-list, the compromise solution, and the weight stability intervals for preference stability of the compromise solution obtained with the initial weights. This method focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. It introduces the multi-criteria ranking index based on the particular measure of ‘‘closeness’’ to the ‘‘ideal’’ solution.

Assuming that each alternative is evaluated according to each criterion function, the compromise ranking could be performed by comparing the measure of closeness to the ideal alternative.

The multi-criteria measure for compromise ranking is developed from the Lp-metric used as an aggregating function in a compromise programming method.

The idea of compromise solution was introduced in MCDM by Po-lung Yn in 1973, and by Milan Zeleny.

The MCDM problem is stated as follows: Determine the best (compromise) solution in multi-criteria sense from the set of J feasible alternatives A1,A2,…AJ, evaluated according to the set of n criterion functions. The input data are the elements fij of the performance (decision) matrix, where fij is the value of the ith criterion function for the alternatives Aj.

2 PRELIMINARIES

2.1 Definition

“Decision making is the process of identifying and selecting a course of action to solve a specific problem.”

2.2 Definition

Decision making involves the selection of a course of action from among two or more possible alternatives in order to arrive at a solution for a given problems.

2.3 Definition

VIKOR method is multi-criteria decision making (MCDM) or multi-criteria decision analysis methods. VIKOR ranks alternatives and determines the solution named compromise that is the closest to the ideal.

3 ALGORITHM OF VIKOR METHOD

- Form a group of decision makers.
- Evaluate the ranking of each criterion according to their significance.
- Normalize the aggregated importance weight for each criterion.
- Form a decision matrix.
- Normalize the decision matrix.
- Construct the weighted normalized decision matrix.
- Calculate the positive and negative ideal solution.
- Determine the distance of each alternative.
- Determine the closeness coefficient and decompose it.
- Rank the alternatives according to their closeness coefficient.

4 THE PROPOSED STEPS OF VIKOR METHOD

According to this method, the best alternative would be the one that is nearest to the positive - ideal solution and farthest from the negative ideal solution. The positive-ideal solution is a solution that maximizes the profit criteria and minimizes the cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the profit criteria. In short, the positive-ideal solution is collected of all best values attainable from the criteria, whereas the negative ideal solution contains all worst values attainable from the criteria.

The calculation procedures of the method are as follows:

A MCDM problem can be concisely expressed in a matrix format, in which columns indicate criteria (attributes) considered in a given problem; and in which rows list the competing alternatives. Specifically, a MCDM problem with m alternatives ($A_1, A_2, \ldots A_m$) that are evaluated by n criteria ($C_1, C_2, \ldots, C_n$) can be viewed as a geometric system with m points in n-dimensional space [1], An element x_{ij} of the matrix indicates the performance rating of the *ith* alternative A_i, with respect to the *jth* criterion C_j, as shown in (1),

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & \cdots & C_n \end{array} \\ \begin{bmatrix} x_{11} & x_{12} & x_{13} & \cdots & x_{1n} \\ x_{21} & x_{22} & x_{23} & \cdots & x_{2n} \\ x_{31} & x_{32} & x_{33} & \cdots & x_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & x_{m3} & \cdots & x_{mn} \end{bmatrix} \end{array} \quad \textbf{(1)}$$

The VIKOR method was introduced as an applicable technique to implement within MCDM. It focuses on ranking and selecting from a set of alternatives in the presence of conflicting criteria. The compromise solution, whose foundation was established by Yu and Zeleny, is a feasible solution, which is the closest to the ideal, and here "compromise" means an agreement established by mutual concessions. The VIKOR method determines the compromise ranking list and the compromise solution by introducing the multi-criteria ranking index based on the particular measure of "closeness" to the "ideal" solution. The multi-criteria measure for compromise ranking is developed from the Lp-metric used as an aggregating function in a compromise programming method. The levels of regret in VIKOR can be defined as:

$$L_{p,i} = \{\textstyle\sum_{j=1}^{n}[\, w_j\,(x_j^* - x_{ij}^-)/(x_j^* - x_j^-)]p\}1/p \qquad ,1 \le p \le \infty \qquad (2)$$

Where i = 1,2,…,m. $L_{1,i}$ is defined as the maximum group utility, and $L_{\infty,i}$ is defined as the minimum individual regret of the opponent. The procedure of VIKOR for ranking alternatives can be described as the following steps:

Step 1: Determine that best $\boldsymbol{x_j^*}$ and the worst $\boldsymbol{x_j^-}$ values of all criterion functions [3], where j = 1, 2,…, n . If the *jth* criterion represents a benefit then $\boldsymbol{x_j^* = \max_i f_{ij}}$, $\boldsymbol{f_j^- = \min_i f_{ij}}$,

Step 2: Compute the S_i (the maximum group utility) and R_i (the minimum individual regret of the opponent) values,

i = 1, 2,…, m , by the relations

$$S_i = L_{1,i} = \textstyle\sum_{j=1}^{n} w_j\,(x_j^* - x_{ij})/\,(x_j^* - x_j^-) \qquad (3)$$

$$R_i = L_{\infty,i} = [\textstyle\sum_{j=1}^{n} w_j\,(x_j^* - x_{ij})\,/\,(x_j^* - x_j^-)] \qquad (4)$$

where w_i is the weight of the *jth* criterion which expresses the relative importance of criteria.

Step 3: Compute the value Q_i , i = 1,2,…,m , by the relation

$$Q_i = v(S_i - S^*)/(S^- - S^*) + (1-v)(\,R_i - R^*)/(R^- - R^*), \qquad (5)$$

Where $S^* = \min_i S_i$, $S^- = \max_i S_i$, $R^* = \min_i R_i$, $R^- = \max_i R_i$ and v is introduced weight of the strategy of S_i and R_i.

Step 4: Rank the alternatives, sorting by the S , R and Q values,in decreasing order. The results are three ranking lists.

Step 5: Propose as a compromise solution the alternative (A′) which is ranked the best by the minimum Q if the following two conditions are satisfied:

C1. "Acceptable advantage":

$Q(A'') - Q(A') \ge DQ$, where A'' is the alternative with second position in the ranking list by Q, $DQ = 1/(m - 1)$ and m is the number of alternatives.

C2. "Acceptable stability in decision making":

Alternative A' must also be the best ranked by S or/and R. This compromise solution is stable within a decision making process, which could be: "voting by majority rule" (when v>0.5 is needed), or "by consensus" $v \approx 0.5$, or "with vote" (v < 0.5). Here, v is the weight of the decision making strategy "the majority of criteria" (or"the maximum group utility"), v=0.5 is used in this paper. If one of the conditions is not satisfied, then a set of compromise solutions is proposed.

Recently, VIKOR has been widely applied for dealing with MCDM problems of various fields, such as location selection, environmental policy, and data envelopment analysis,

5 INFORMATION ENTROPY WEIGHT

The weight of the criterion reflects its importance in MCDM. In this paper, an objective weight is applied; named Information Entropy Weight (IEW) based on the information entropy of raw data. Based on the decision matrix (D) shown in (1), Range standardization is done to transform different scales and units among various criteria into common measurable units in order to compare their weights.

$$x_{ij}^{'} = \frac{x_{ij} - \min_{1 \le j \le n} x_{ij}}{\max_{1 \le j \le n} x_{ij} - \min_{1 \le j \le n} x_{ij}} \quad (6)$$

$D^{'}=X^{'}_{m \times n}$ is the matrix after range standardization; max X_{ij}, min X_{ij} are the maximum and the minimum values of the criterion (j) respectively, all values in $D^{'}$are (0≤ $x_{ij}^{'}$ ≤ 1). So, according to the normalized matrix $D^{'}=X^{'}_{m \times n}$ the information entropy is calculated as shown in the following steps, first in order to avoid the insignificance of ln f_{ij} in (8) f_{ij} is stipulated as shown in (7):

$$f_{ij} = \frac{1+x_{ij}^{'}}{\sum_{i=1}^{m} 1+x_{ij}^{'}} \quad (7)$$

$$H_j = -\sum_{i=1}^{m} f_{ij} In f_{ij} , \quad i=1,2,\ldots\ldots\ldots m;\ j=1,2,\ldots\ldots\ldots n \quad (8)$$

After calculating the variation degree (H_j), the deviation degree of the criterion (j) noted by (G_j) is computed as in (9)

$$G_j = 1 - H_j ,\ j=1,2,\ldots\ldots\ldots n \quad (9)$$

It is obvious that (G_j) is greater if the value of (H_j) is smaller, consequently if the (G_j) is higher, the information entropy (H_j) is lower, which indicates that the more the information criterion (j). The weight (W_j) of the criterion (j) is defined as:

$$W_j = \frac{G_j}{\sum_{j=1}^{n} G_j} = \frac{1-H_j}{n-\sum_{j=1}^{n} H_j} \quad (10)$$

Where j=1,2,.....,n.

6 NUMERICAL EXAMPLE

We will proceed this paper to take the decision making in usage of social media in different age groups by VIKOR method. We have taken different types of social media, they are YouTube, Facebook, Snapchat, Instagram and Twitter. The social media which is used in various age group people, such as 18-29, 30-49, 50-64, 65+.

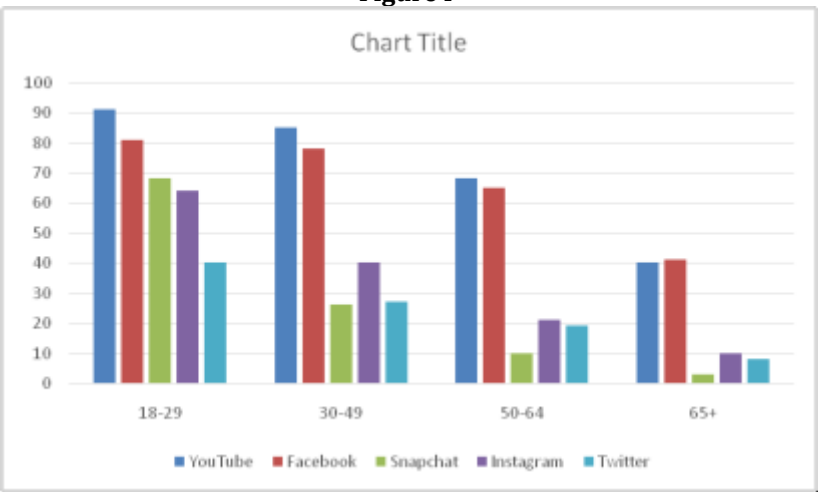
Table I

Age	YouTube	Facebook	Snapchat	Instagram	Twitter
18-29	91	81	68	64	40
30-49	85	78	26	40	27
50-64	68	65	10	21	19
65+	40	41	3	10	8

Decision matrix

The figure shows the usage data of social media in different age groups.

Figure I



The Information Entropy Weight objective method will be applied in this problem Table II, illustrates the range standardization done to decision matrix as in (6).

Table II

Range standardization decision matrix

Age	YouTube	Facebook	Snapchat	Instagram	Twitter
18-29	1	0.80	0.54	0.47	0
30-49	1	0.88	0	0.23	0.01
50-64	1	0.94	0	0.18	0.15
65+	0.97	1	0	0.18	0.13

Table III shows the stipulation found in (7).

Table III

Sipulated matrix

Age	YouTube	Facebook	Snapchat	Instagram	Twitter
18-29	0.2509	0.2362	1	0.2905	0
30-49	0.2509	0.2467	0	0.2430	0.3069
50-64	0.2509	0.2545	0	0.2332	0.3495
65+	0.2471	0.2624	0	0.2332	0.3434

Table IV shows the values of the variation degree (H_j), the deviation degree (G_j), and the weight assigned to each criterion (W_j) based on information entropy as in (8), (9), and (10)

Table IV

Weights assigned to Criteria

	H_j	G_j	W_j
YouTube	0.602016	0.397984	0.146332
Facebook	0.601697	0.398303	0.146449
Snapchat	0	1	0.367683
Instagram	0.600143	0.399857	0.147020
Twitter	0.476411	0.523589	0.192514

By using the procedure of VIKOR, we can calculate the S, R and Q values as shown in Table III to derive the preference ranking of the alternatives. We should select the first and the second age group people for the using social media. Both age group people have the minimum S, R, and Q values, also the two conditions mentioned earlier in section 2 are satisfied.

Table V
Raking list and scores

	You Tube	Facebook	Snapchat	Instagram	Twitter	S_i	R_i	Q_i	Rank
18-29	0	0	0	0	0	0	0	0	1
30-49	0.0172	0.0109	0.2375	0.0653	0.0782	0.4091	0.2375	0.52763	2
50-64	0.0659	0.0585	0.3280	0.1170	0.1263	0.6957	0.3280	0.79404	3
65+	0.1463	0.1464	0.3676	0.1470	0.1925	0.9998	0.3676	1	4

CONCLUSION

A VIKOR method combined to Information Entropy Weight method is presented to solve the MCDM problem. A social media usage of various age group people of a new manner is introduced. The VIKOR method is introduced to solve this MCDM problem based on the classical utility function. It might be combined to other techniques in further research. The MCDM problem should be reformulated and solved if any parameter or alternative is added or deleted because of its sensitivity to any changes.

Social media is a web-based technology to facilitate social interaction between a large group of people through some type of network . Social media is growing rapidly and becoming a vital part of everyday life, because of the latest technological revolution.

Social media platforms allow users to have conversation, share information and create web content. There are many forms of social media including blogs, micro-blogs, wikis, social networking sites, photo-sharing sites, instant messaging, video-sharing sites, podcasts, widgets, virtual worlds, and more.

Social media is becoming an integral part of life. Most traditional online media include social components, such as comment fields for users. In business, social media is used to market products, promote brands, connect to current customers and foster new business.

Social media plays an important role in every student's life. It is easier and convenient to access information, provide information and communicate via social media. Teachers and students are connected to each other and can make good use of these platforms for the working of their education.

People who are addicted to social media may experience negative side effects such as eye strain, social withdrawal or lack of sleep, stress. If you spend your time researching problems or arguing with people, you may experience stress, which can have a negative impact on your health.

As well as providing social support, social media sites can also provide medical support. Although there are many positive aspects of social media, the negative effects on children and adolescents are also numerous. Social media can affect the mental health of teens.

Researchers have found that using social media obsessively causes more than just anxiety. In fact, testing has found that using too much internet can cause depression, Attention Deficit Hyperactivity Disorder (ADHD), impulsive disorder, problems with mental functioning, paranoia, and loneliness.

Social networking plays a crucial role in socialization and provides a platform in building relationships. Socialization and social networking are deeply rooted in human psychology as both rely on aspects of a humanistic approach to affect its core audience.

Excessive cellphone use may cause anxiety, experts warn. Spending too much time on your phone may be causing you to feel stress and anxiety, experts are warning. Most people experience an emotional response that floods their body with stress hormones when they hear their phone go off, according to Cheever.

Media plays a very important role and has influenced virtually in every aspect of our lives. Newspaper, Magazines, Radio, Television and Internet are the different types of media. It greatly affects our lives because media has the power to influence our thoughts.

Research has found that how media can influence children's beliefs and behaviors in terms of violence and aggression, sex, substance abuse, obesity and eating disorders.

Impact of social media on self-esteem, has been linked to higher levels of loneliness, envy, anxiety, depression, narcissism and decreased social skills.

From the analysis, we concluded that the 18-29 age group people were mostly used social media. We share some social issues in social media it can be shared and helped by affecting people but we share our personal things it would be misled by some people. It may be both advantage and disadvantage, but can use in a good manner.

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DISTANCE - 2 DOMINATING SETS OF GRAPHS IN NETWORKS

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ABSTRACT: *A graph G=(V,E) consists of a set V of vertices and E of edges. A vertex v in a graph G is said to be dominate itself and each of its neighbors . We say with other words that v dominates the vertices of its closed neighborhoods. The aim of the paper is to impart the importance of graph theoretical concepts and the applications of distance – 2 domination in graphs to various real life situations in the areas of science and engineering. A set D is a distance - 2 dominating set if for every vertex* $u \varepsilon V-D$, $d(u, D) \leq 2$ *and is denoted by* $\gamma \leq 2(G)$. *Also this paper explores mainly on the applications of distance - 2 dominating sets in networks. Distance - 2 dominating sets are identified in School Bus Routing, Radio Stations, Communication Networks . We study on the concept of distance -2 dominating sets in graphs in networks.*

Keywords:

INTRODUCTION

Domination in graphs has been an extensively researched branch of graph theory. Graph theory is one of the most flourishing branches of modern mathematics and computer applications. The last 30 years have witnessed spectacular growth of graph theory due to its wide applications to discrete optimization problems, combinatorial problems and classical algebraic problems .

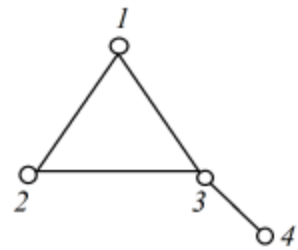
It has a very wide range of applications to many fields like engineering, physical, social and biological sciences; linguistics etc., the theory of domination has been the nucleus of research activity in graph theory in recent times.

This is largely due to a variety of new parameters that can be developed from the basic definition of domination.The rigorous study of dominating sets in graph theory began around 1960, even though the subject has historical roots dating back to 1862 when de jaenisch studied the problems of determining the minimum number of queens which are necessary to cover or dominate a n*n chessboard.

In 1958,Berge defined the concept of the domination number of a graph ,calling this as "Coefficient of External Stability". In 1962, Ore used the name "dominating set" and "domination number " for the same concept.In 1977 Cockayne and Hedetniemi made an interesting and extensive survey of the results know at that time about dominating sets in graphs.

1. DEFINITIONS

1.1 Dominating set *:A* set *D* of vertices in a graph G,is a dominating set, if every vertex not in *D* (i.e.) (every vertex *in V — D)* is adjacent to at least one vertex in *D.*



D1 = {1} D2 = {1,2}, D3 = {3},D4 = {4,1}, D5 = {1,2,3,4}.

*D1*and *D2* are not dominating sets;D3, D4, D5 are dominating sets.

1.2 Minimal dominating set:

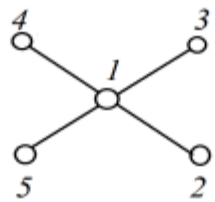
A dominating set *D* is called a minimal dominating set, if for every vertex *v,D — {v}* isnot a dominating set.

1.3 ***Minimum dominating set:***

A dominating set *D* is called a minimum dominating set, if *D* consists ofminimum number of vertices among all dominating sets.

1.4 *Domination number γ* (G):

The number of vertices in a minimum dominating set is defined as the dominationnumber of a graph *G,* and it is denoted by (G).



D1 = {1},D2 = {2,3,4,5}.
D1 and *D2* are minimal dominating sets.
D2= {1} is the minimum dominating set since it has minimum number of vertices thanD2.
(G) = 1 is the domination number *of G.*

Theorem 1.1

A dominating set *D* is minimal dominating set if and only if for each vertex *v* in D, one ofthe following conditions holds; v is an isolated vertex of D; There exists a vertex u in*V — D* such that intersection *of N{u} and D = {v}.*

Proof:

If each vertex v in D has at least one of the properties then D –{v} is not a dominating set of G.
Consequently, D is a minimal dominating set of G.
Assume that D is a minimal dominating set of G.
Then for each vεD, the set D - {v} is not a dominating set of G.
Hence there is a vertex u in V(G)-(D-{v}) that is adjacent to no vertex of D-{v}.
If u=v then v is adjacent to no vertex of D.

Theorem 1.2

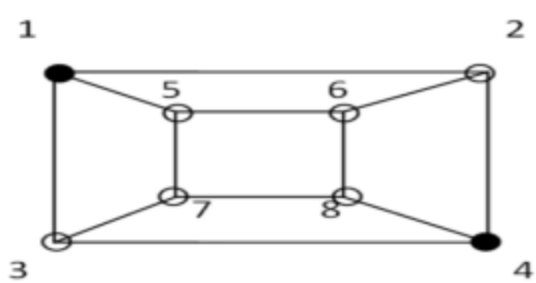
Let G be a graph without isolated vertices. If D is a minimal dominating set, then V — D isa dominating set.

Proof:

Let vεD. Then v has at least one of the two properties of the theorem 1.1.
Suppose first that there exist a vertex u in V(G)-D such that N{u} and D = {v}.
Hence v is adjacent to some vertex in V(G)-D.
Suppose next that v is adjacent to no vertex in D.
Then v is isolated vertex of the sub graph D.
Since v is not isolated in G, the vertex v is adjacent to some vertex of V(G)-D.
Thus V(G)-D is a dominating set of G.

2. DISTANCE -2 DOMINATING SET

A set D of vertices in a graph G=(V,E) is a distance-2 dominating set if every vertex in V-D is within distance 2 of atleast one vertex inD. T he distance 2 domination number $\gamma_{\leq 2}$ (G) of G equals to the minimum cardinality of a distance to dominating set in G.



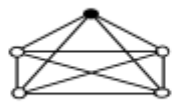
Graph G

Here $D = \{1,4\}$, $\gamma_{\leq 2}(G) = 2$

2.1 Observation

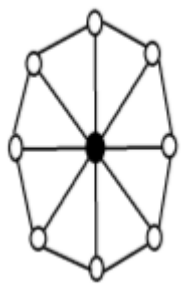
1. For any Complete graph K_n , for n ≥ 2

$\gamma_{\leq 2}(K_n) = 1$



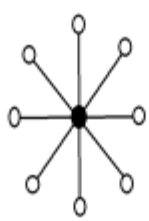
1. For any Wheel graph W_n , for n ≥ 4

$\gamma_{\leq 2}(W_n) = 1$



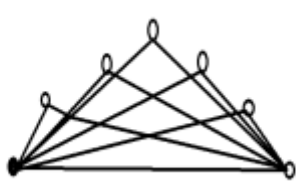
2. For any Star $K_{1,n}$, for n ≥ 2

$\gamma_{\leq 2}(K_{1,n}) = 1$



3.For any Book graph B_n , for n ≥ 3

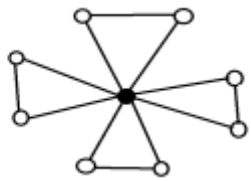
$\gamma_{\leq 2}(B_n) = 1$



4.For any Complete Bipartite graph $K_{n,m}$, for n,m≥ 1

$\gamma_{\leq 2}(K_{n,m}) = 1$

1. For any Friendship graph F_n , for n ≥ 2
   $\gamma_{\leq 2}(F_n) = 1$



2.2 Proposition

1. For any graph G , $\gamma_{\leq 2}(G) \leq \gamma(G)$.

The converse of the above proposition need not be true $\gamma_{\leq 2}$

2. For a Grid graph $G_{2,k}$, for k $fork \geq 1$,

$\gamma_{\leq 2}(G_{2,k}) = [\frac{2+k}{3}]$

3. Let D be a distance - 2 dominating set of a graph G. Then D is a minimal distance - 2 dominating set if and only if each vertex uεD satisfies at least one of the following conditions:

(a). there exists a vertex vε V(G) – D for which $N \leq 2(v) \cap D = \{u\}$.

(b). d(u, w) > 2 for every vertex wε D-{u}.

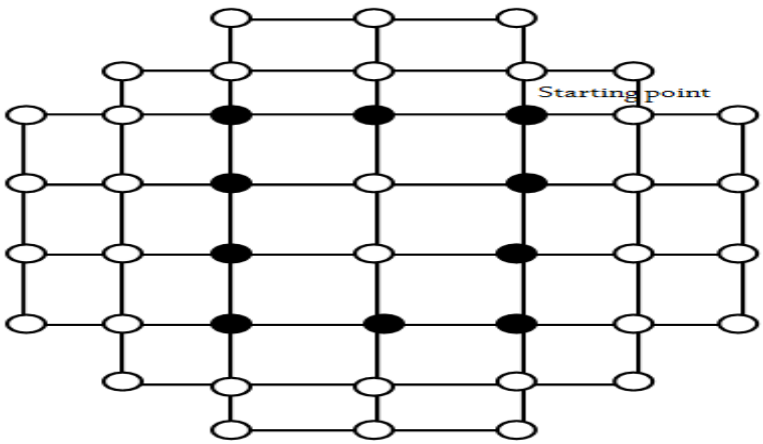
3. Applications of distance - 2 dominating sets

When we extend the concept of dominating sets to distance - 2 dominating sets, there are more useful models to many real- world problems. Indeed, much of the motivation for the study of domination arises from problems involving locating optimally a hospital, police station, fire station, or any other emergency service facility.

3.1 Bus Routing

Nowadays, almost all Colleges operate College buses for transporting students for to and fro services. Among many points, three important points to be noted are (i) The running time of a bus between College and its terminus (ii) Maximum number of students on a bus at any time and (iii) the maximum distance a student has to walk to board a college bus. Consider a street map of a city shown in fig where each edge represents one city block. Let us assume that the college is

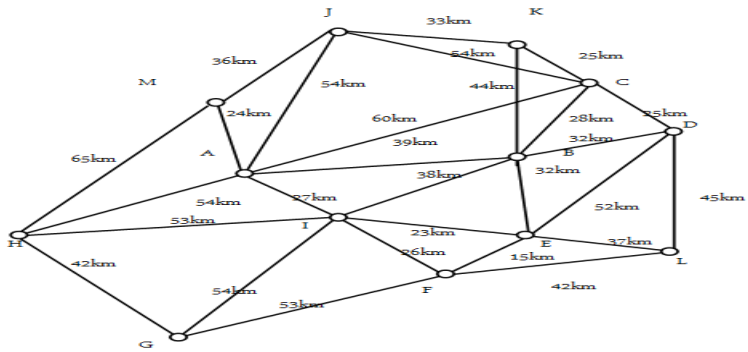
located at the vertex starting point and the management committee of the college decides that no student shall walk more than two blocks to board a college bus. Construct a route for a college bus that leaves the college, gets within two blocks of every student that uses the college bus and returns to the college. Clearly this bus route forms a distance - 2 dominating set.

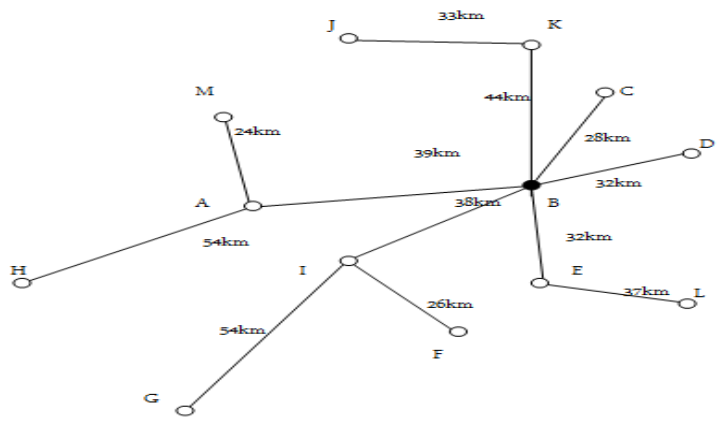


3.2 Radio Stations

Suppose that we have a collection of small villages in a remote part of the world. We would like to locate radio stations in some of these villages so that messages can be broadcasted to all the villages in the region. But since the installations of radio stations are costly, we want to locate as few as possible which can cover all other villages. Let each village be represented by a vertex. An edge between two villages is labeled with the distance, say in kilometers. Let us assume that a radio station has a broadcast range of hundred

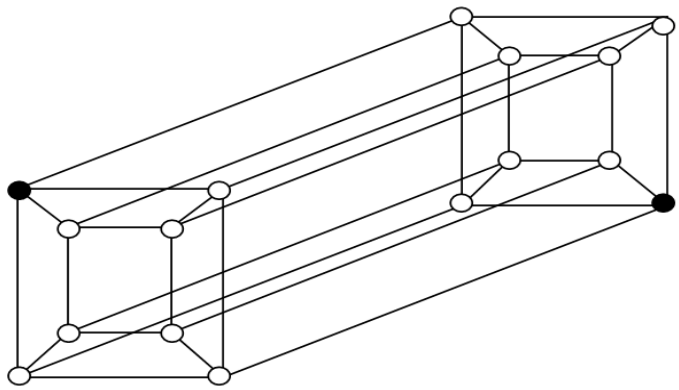
kilometers. In this case we seek a distance - 2 dominating set among all the vertices within the distance of 100 kilometers. Clearly this fig gives the distance - 2 dominating set.





3.3 Computer Communication Networks

The distance - 2 dominating set plays an important role in computer and communication networks to route the information between the nodes. We consider a computer network modeled by a Hyper cube. The vertices of the Hyper cube represents computer and edges represent direct communication link between two computers. So, in this model we have 16 computers or processors and each processor can pass information to the processor to which it is directly connected. Our problem is to collect information from all processors and we would like to do it relatively often and relatively fast. So, we identify a small set of processors called collecting processors and ask each processor to send its information to one of the small sets of collecting processors. We assume that at most a two – unit delay between the time a processor sends its information and the time it arrives at a nearest collector is allowed. In this case, we have to find a distance – 2 dominating set of all processors. The set of vertices {marked in dark} forms a distance 2 dominating set in the hypercube network in fig.



3.4 Locating Radar Stations Problem

The problem was discussed by Berge . A number of strategic locations are to be kept under surveillance. The goal is to locate a radar for the surveillance at as few of these locations as possible. How a set of locations in which the radar stations are to be placed can be determined.

3.5 Facility Location Problems

The dominating sets in graphs are natural models for facility location problems in operational research. Facility location problems are concerned with the location of one or more facilities in a way that optimizes a certain objective such as minimizing transportation cost, providing equitable service to customers and capturing the largest market share.

4. CONCLUTION:

The main aim of this paper is to present the importance of graph theoretical ideas in various areas of Science & Engineering for researches that they can use Domination in graph theoretical concepts for the research. An overview is presented especially to project the idea of graph theory. So, the graph theory section of each paper is given importance than to the other sections. Researches may get some information related to graph theory and its applications in various field and can get some ideas related to their field of research.

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FUZZY LOGIC AND ITS APPLICATIONS

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ABSTRACT: *Thinking and reasoning are the part of nature. Study of reasoning is nothing but analyzing the human thought processes and reasoning methodologies. The idea of Fuzzy Logic is same as the feeling of human. Fuzzy Logic is a form of many valued logic or probabilistic logic; it deals with reasoning that is approximate rather than fixed and exact. In this work, we study some applications of Fuzzy Logic in real life. Fuzzy Logic is mostly used in Control systems with the use of Rule base. We also have two types of Fuzzy Inference System for controlling the Electronic system and devices.*

Keywords: *Fuzzy Logic, Fuzzy Inference System (FIS), Rule Base.*

I.INTRODUCTION

In the last part of 19th century, George Boole (1815-1864), an Irish Mathematician and logician, started a systematic study of logic. It is called a Boolean Logic.

In 1965 Lotfi Zadeh, published his famous paper "Fuzzy sets". Zadeh extended the work on possibility theory into a formal system of mathematical logic, and introduced a new concept for applying natural language terms. This new logic for representing and manipulating fuzzy terms was called fuzzy logic, and Zadeh became the Master of fuzzy logic.

Fuzzy Logic was formulated by Lofti A Zadeh of the University of California at Berkeley in the mid 1960's.

Binary or Boolean logic simply gives the statement is true i.e.1 or false i.e.0. But Fuzzy Logic includes 0 and 1 and tell the statement is how much true or false using the in between values of 0 and 1.Fuzzy Logic defines operations for modifying sets and allows elements to partially belong to the set, which offers a lot more flexibility.

II. PRELIMINARIES

2.1 Fuzzy set

If X is a collection of objects denoted generically by x, then a fuzzy set A in X is a set of ordered pairs:

$A=\{(x, \mu_A(x))/ x \in X\}$

$\mu_A(x)$ is called the membership function or grade of membership of x in A that maps X to the membership space M.

2.2 Membership Function

Let X denote a universal set. Then, the membership function μ_A by which a fuzzy set A is usually defined has the form

$$\mu_A: X \rightarrow [0, 1]$$

Where [0,1] denotes the interval of real numbers from 0 to 1.

$$0 \leq \mu_A \leq 1$$

2.3 Fuzzy Inference System

Fuzzy Inference System have been successfully applied in variety of fields such as automatic control, data classification, decision analysis, expert systems etc. Fuzzy Inference is the process of mapping a given set of inputs to an output using fuzzy logic defined by the fuzzy 'If-Then' rule database. The mapping provides a basis for computing the decisions.

There are two types of Fuzzy Inference System that can be implemented

- Mamdani Fuzzy Inference System
- Sugeno Fuzzy Inference System

The Fuzzy Inference process have the five steps

1. **Fuzzy Inputs**

   The first step to take inputs and determine the degree to which they belong to each of the appropriate fuzzy sets via membership functions.

2. **Apply Fuzzy Operators**

Once the inputs have been fuzzified, we know the degree to which each part of the antecedent has been satisfied for each rule. If a given rule has more than one part, the fuzzy logical operators are applied to evaluate the composite firing strength of the rule.

3. **Apply the Implication Method**

The implication method is defined as the shaping of the output membership functions on the basis of the firing strength of the rule. The input to the implication process is a single number given by the antecedent, and the output is a fuzzy set. Two commonly used methods of implication are the minimum and the product.

4. **Aggregate all Outputs**

Aggregation is a process whereby the outputs of each rule are unified. Aggregation occurs only once for each output variable. The input to the aggregation process is the truncated output fuzzy sets returned by the implication process for each rule. The output of the aggregation process is the combined output fuzzy set.

5. **Defuzzification**

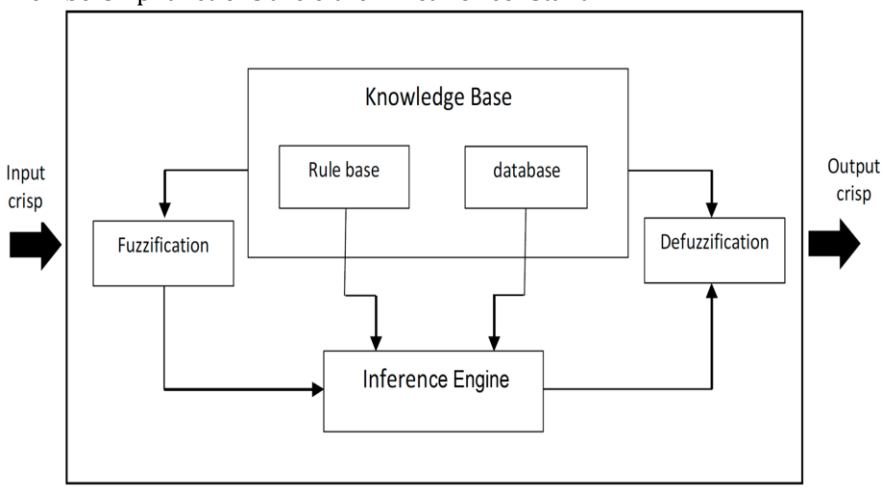
The input for the defuzzification process is a fuzzy set and the output of the defuzzification process is a crisp value obtained by using some defuzzification method such as centroid, height, or maximum.

2.4 Mamdani Fuzzy Inference System

Mamdani Fuzzy Inference System is the most commonly seen fuzzy methodology. It was among the first control systems built using Fuzzy set theory. It was proposed in 1975 by Ibrahim Mamdani as an attempt to control a system engine.

2.5 Sugeno Fuzzy Inference System

Sugeno or Takagi-Sugeno-Kang(TSK) method of Fuzzy Inference, introduced in 1985, is similar to the Mamdani method in many aspects. The main difference between Mamdani and Sugeno is that the Sugeno output membership functions are either linear or constant.



III. FUZZY RULE BASE

Fuzzy Logic usually uses IF-THEN rules. Rule Construction is,

IF variable IS property THEN action

For example,

A simple temperature regulator uses for a fan might look like this:

IF temperature IS very cold THEN stop fan
IF temperature IS cold THEN turn down fan
IF temperature IS normal THEN maintain level
IF temperature IS hot THEN speed up fan

IV.FUZZY CONTROL FOR WASHING MACHINE

When one uses a washing machine, the person generally select the length of wash time based on the amount of clothes he/she wish to wash and the type and degree of dirt cloths have. To automate this process, we use sensors to detect these parameters.

The wash time is then determined from this data.

The two inputs are:

1. Degree of dirt
2. Type of dirt

RULE BASE FOR WASHING MACHINE

1. If dirtness of clothes is Large and type of dirt is Greasy then wash time is Very Long;
2. If dirtness of clothes is Medium and type of dirt is Greasy then wash time is Long;
3. If dirtness of clothes is Small and type of dirt is Greasy then wash time is Long;
4. If dirtness of clothes is Large and type of dirt is Medium then wash time is Long;
5. If dirtness of clothes is Medium and type of dirt is Medium then wash time is Medium;
6. If dirtness of clothes is Small and type of dirt is Medium then wash time is Medium;
7. If dirtness of clothes is Large and type of dirt is Not Greasy then wash time is Medium;
8. If dirtness of clothes is Medium and type of dirt is Not Greasy then wash time is Short;
9. If dirtness of clothes is Small and type of dirt is Not Greasy then wash time is Very Short

V. FUZZY CONTROL FOR AIR CONDITIONING SYSTEM

Air Conditioning is the process of removing heat from a confined space, thus cooling the air and removing humidity. Air Conditioning shortly called AC.The first input temperature is taken to be in ranges of 0ºC to 45ºC and have four triangular membership functions namely, 1) very low, 2) low, 3) high, and 4) very high

The second input Humidity is taken in the range from 0% to 100% and also have four membership functions namely, 1) dry, 2) comfortable, 3) humid, and 4) sticky.

The output Compressor speed is taken in percentage in range from 0% to 100% and have four triangular membership functions namely, 1) off, 2) low, 3) medium, and 4) fast.

RULE BASE FOR AC

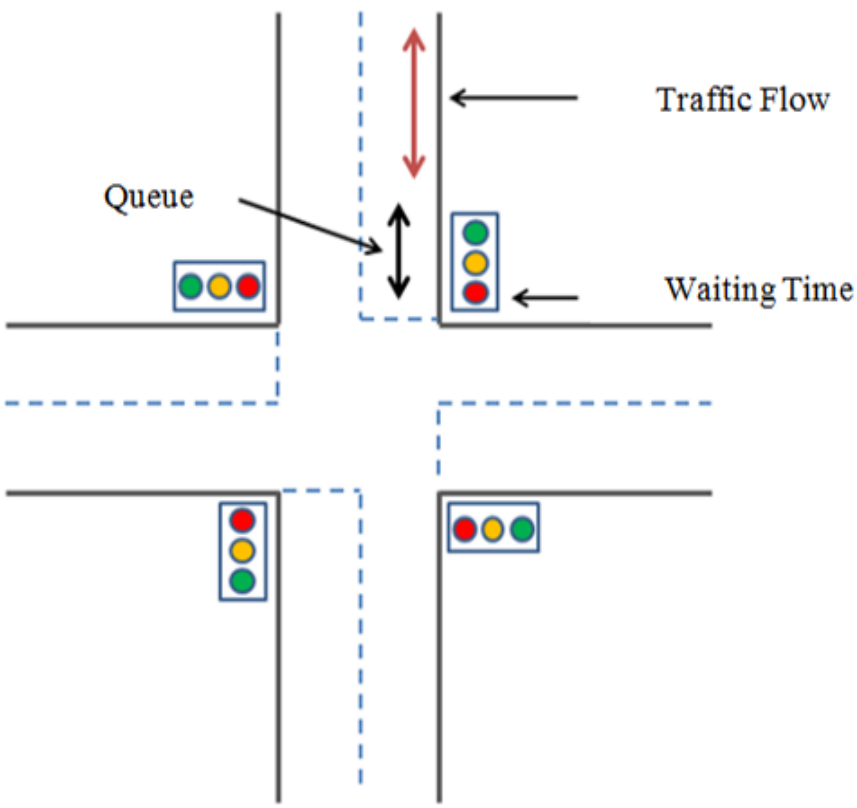
Rules	Temperature	Humidity	Compressor speed
1	Very Low	Dry	Off
2	Very Low	Comfortable	Off
3	Very Low	Humid	Off
4	Very Low	Sticky	Low
5	Low	Dry	Off
6	Low	Comfortable	Off
7	Low	Humid	Low
8	Low	Sticky	Medium
9	High	Dry	Low
10	High	Comfortable	Medium
11	High	Humid	Fast
12	High	Sticky	Fast
13	Very High	Dry	Medium

14	Very High	Comfortable	Fast
15	Very High	Humid	Fast
16	Very High	Sticky	Fast

VI.FUZZY LOGIC IN TRAFFIC CONTROL

Traffic control is an important problem which must be resolved. High traffic control is caused by the number of vehicles increasing. The numbers of queues which pass a crossroad often cause traffic control. A lot of intersections have used traffic lights to regulate it. The traffic light control system is developed by considering the traffic flow of vehicle which comes to the intersection. The traffic flow can be obtained from the number of arriving vehicles per minute.

DESIGN OF TRAFFIC CONTROL SYSTEM



INPUTS AND OUTPUTS OF FUZZY TRAFFIC CONTROL

The input in the proposed system consists of three variables which are

1. The number of queue (NQ)
2. Traffic flow of vehicles (TF) per minute
3. Waiting time (WT) in seconds.

The output of the system is green time duration (GT). The first input, NQ is divided into six membership functions. Neutral (N), very small (VS), small (S), medium (M), big (B) and very big (VB) with range [0:80].

The second input, TF is divided into three membership functions. Small (S), medium (M) and big (B) with range [0:20].

The third input, WT is divided into three membership functions. Small (S), Medium (M) and big (B) with range [0:20].

The output, GT in a singleton form and it is divided into six membership functions. Neutral (N), very small (VS), small (S), medium (M), big (B) and very big (VB) with range [0:50].

FUZZY RULES FOR TRAFFIC CONTROL SYSTEM

1. If NQ is N then GT is N
2. If NQ is VS and TF is S then GT is N
3. If NQ is VS and TF is M then GT is N
4. If NQ is VS and TF is B then GT is N
5. If NQ is S and TF is S then GT is VS
6. If NQ is S and TF is M then GT is VS
7. If NQ is S and WT is S and TF is B then GT is VS
8. If NQ is S and WT is M and TF is B then GT is S
9. If NQ is S and WT is B and TF is B then GT is S
10. If NQ is M and TF is S then GT is S
11. If NQ is M and WT is S and TF is M then GT is S
12. If NQ is M and WT is M and TF is M then GT is M
13. If NQ is M and WT is B and TF is M then GT is M
14. If NQ is M and WT is S and TF is B then GT is M
15. If NQ is M and WT is M and TF is B then GT is M
16. If NQ is M and WT is B and TF is B then GT is B
17. If NQ is B and TF is S then GT is M
18. If NQ is B and WT is S and TF is M then GT is M
19. If NQ is B and WT is M and TF is M then GT is M
20. If NQ is B and WT is B and TF is M then GT is B
21. If NQ is B and WT is S and TF is B then GT is B
22. If NQ is B and WT is M and TF is B then GT is B
23. If NQ is B and WT is B and TF is B then GT is VB
24. If NQ is VB and TF is S then GT is B
25. If NQ is VB and WT is S and TF is M then GT is B
26. If NQ is VB and WT is M and TF is M then GT is B
27. If NQ is VB and WT is B and TF is M then GT is VB
28. If NQ is VB and WT is S and TF is B then GT is B
29. If NQ is VB and WT is M and TF is B then GT is VB
30. If NQ is VB and WT is B and TF is B then GT is VB

VII.CONCLUSION

This study has examined the performance of Fuzzy Logic in Control Systems. It is used to know how the control system works with Fuzzy Logic controllers and Fuzzy Inference Systems.

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STUDY ON MEAN LABELING OF BULL GRAPH

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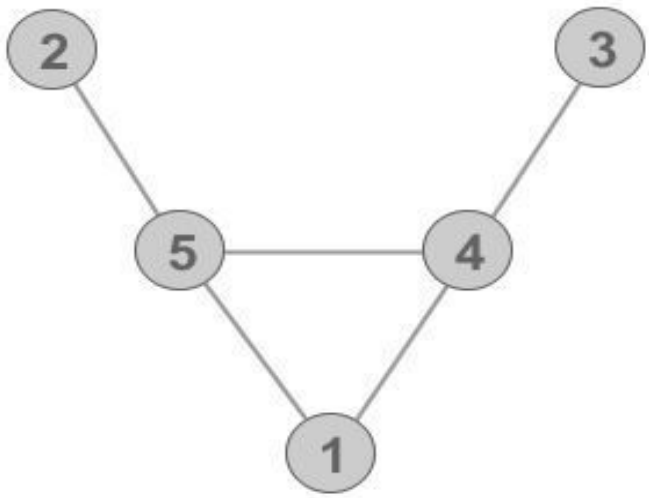
ABSTRACT: *A graph G with p vertices and q edges is a mean graph if there is an injective function f from the vertices of G to {0, 1, 2,....q} such that when each edge* uv *is labeled with* $f(u)+f(v)/2$ *if* $f(u)+f(v)$ *is even and* $f(u)+f(v)+1/2$ *if* $f(u)+f(v)$ *is odd then the resulting edges are distinct. In this paper we investigate mean labeling of shadow graph of bistar and comb and splitting graph of comb.*

Keywords:

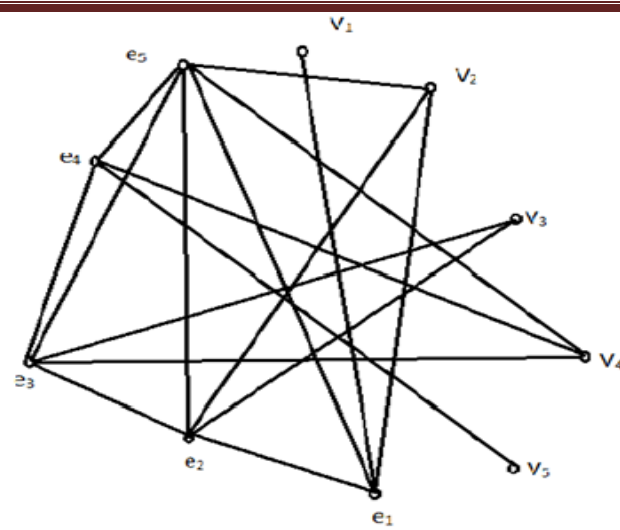
Introduction:
By a graph $G = (V(G), E(G))$ with p vertices and q edges we mean a simple, connected and undirected graph. In this paper a brief summary of definitions and other information is given in order to maintain compactness. In this paper we investigate mean labeling of shadow graph of bistar and comb and splitting graph of comb.

Bull Graph:
By a graph here we mean a finite undirected graph without loops and multiple edges. The Bull Graph is a planar undirected graph with 5 vertices and 5 edges, in the form of a triangle with two disjoint pendant edges

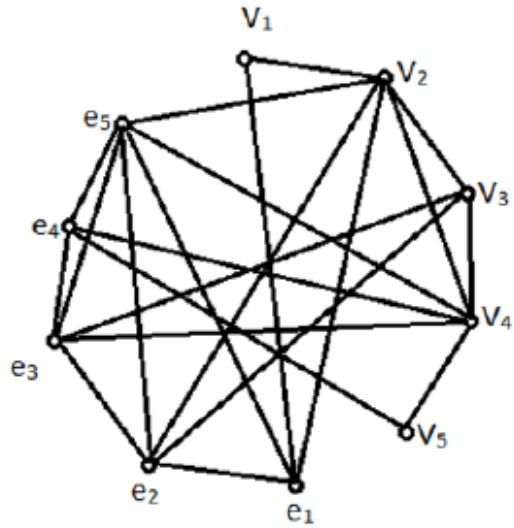


The Bull Graph was introduced by E. W. Weisstein [16] It is a simple graph with radius 2, diameter 3, chromatic number 3 and chromatic index 3 whose name derives from its resemblance to a schematic illustration of a bull or ram (whose face is the triangle and horns are the graph's two bridges).

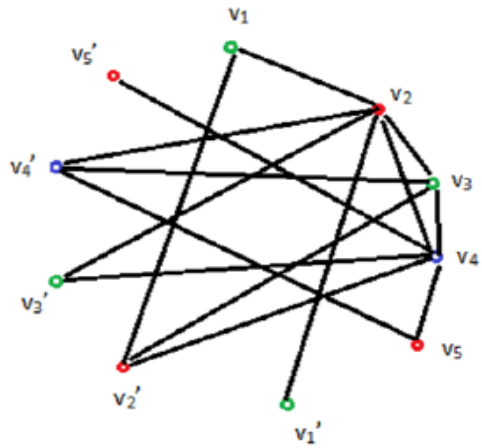
Definition*: 1* *The middle graph* M (G) of a graph G is the graph whose vertex set is V(G)UE (G) and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and other is an edge incident on it.



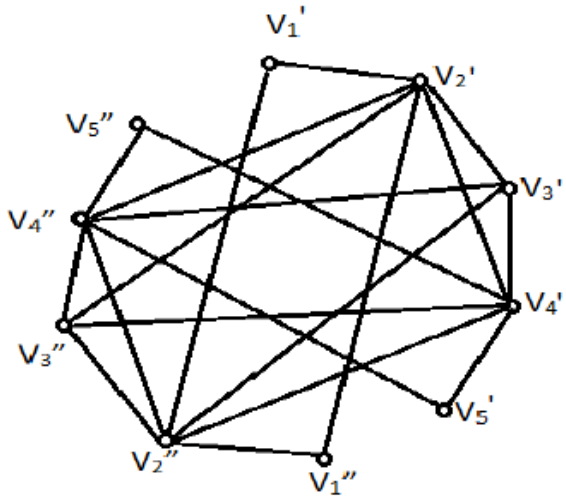
Definition:*2* *The total graph* T (G) of a graph G is the graph whose vertex set is V (G) U E(G) and two vertices are adjacent whenever they are either adjacent or incident in G.



Definition:3 *The splitting graph* S'(G) of a graph G is obtained by adding a new vertex v' corresponding to each vertex v of G such that N (v) =N (v') where N (v) and N (v') are the neighborhood sets of v and v' respectively.



Definition*: 4* *The Shadow Graph* D2(G) of a connected graph G is constructed by taking two copies of G, say G' and G''. Join each vertex u' in G' to the neighbors of the corresponding vertex u'' in G''.



Theorem 1: The graph $D2(B\,n, n)$ has mean labeling.

Proof: Consider two copies of $B\,n, n$. Let $v1, v2, v1, j, v2, j, 1 \le j \le n$ be the vertices of first copy of $B\,n, n$ Let $u1, u2, u1, j, u2, j, 1 \le j \le n$ be the vertices of second copy of Bn, n where $v1, v2$ and $u1, u2$ are the respective apex vertices.

Let $D2\ B\,n, n$ b the shadow graph of the two copies of $B\,n, n$.

Define f:($D2\ \ Bn,n \rightarrow$ 0,1,2,.....q by

$f\ v1 = 0$

$f\ v2 = 8n + 1$

$f\ u1 = 4n$

$f\ u2 = 8n + 3$

$f\ v1, = 4j - 3\ if\ 1 \le j \le n$

$f\ v2, = 4j\ if\ 1 \le j \le n-1$

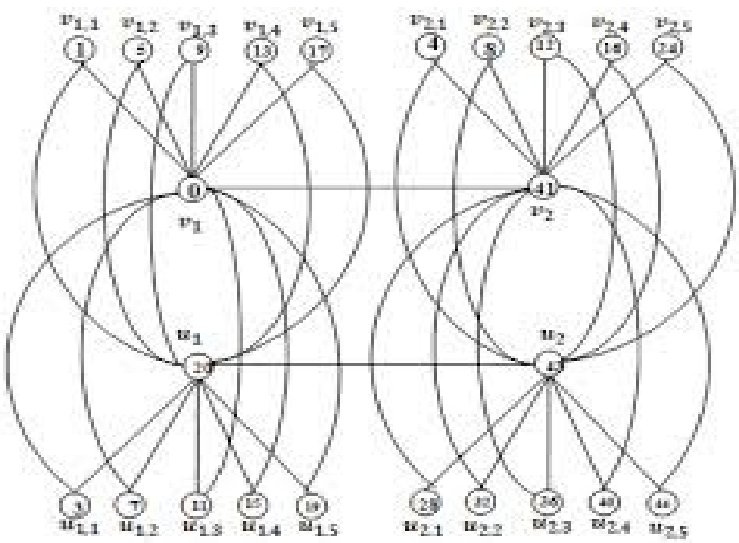
$f\ v2n = 4n+4$

$f\ u1, = 4j - 1\ if\ 1 \le j \le n$

$f\ u2, = 4(n + j + 1)\ if\ 1 \le j \le n$

Thus the induced edge labels are distinct. Hence the graph $D2(B\,n, n)$ has mean labeling.

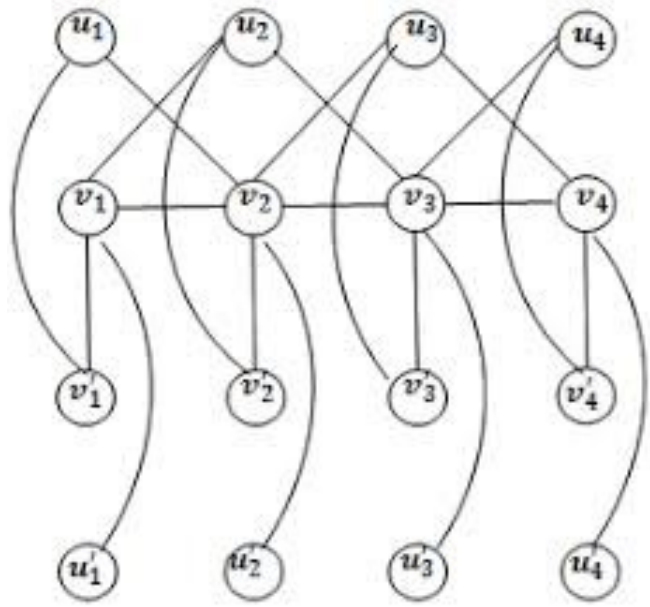
Example:1



Mean labeling for $D2(B5,5)$

Theorem:2 The split graph of comb has mean labeling.

Proof: Let $\{vi, 1 \le i \le n\}$ and $\{vi', 1 \le i \le n\}$ be the vertices of comb in which $\{vi', 1 \le i \le n\}$ are the pendent vertices Let $\{ui, 1 \le i \le n\}$ and $\{ui', 1 \le i \le n\}$ be the newly added vertices.



The ordinary labeling for split graph of comb.

Define $f: v\,G \to \{0,1,2,....q\}$ by

$f\,v = 6i - 4\ if\ i\ is\ odd$

$= 6i - 3\ if\ i\ is\ even$

$f\,v1' = 1$

$f\,v3' = 15$

Let $f\,vi' = 6i - 3\ if\ i\ is\ odd\ i \ne 1,3$

$= 6i - 4\ if\ i\ is\ even$

$f\,ui = 6i - 6\ if\ i\ is\ odd$

$= 6i - 7\ if\ i\ is\ even$

$f\,u1' = 3$

$f\,u3' = 11$

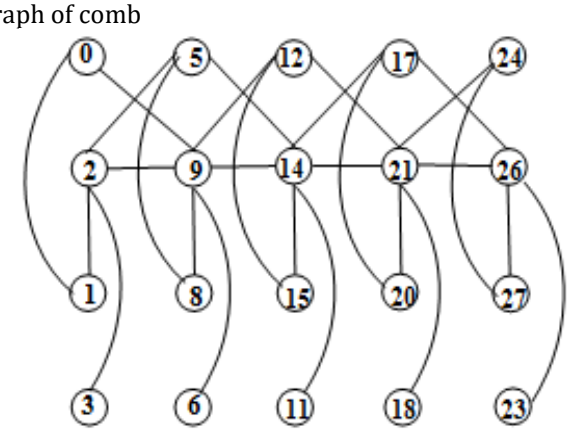
$f\,ui' = 6i - 7\ if\ i\ is\ oddi \ne 1,3$

$= 6i - 6\ if\ i\ is\ even$

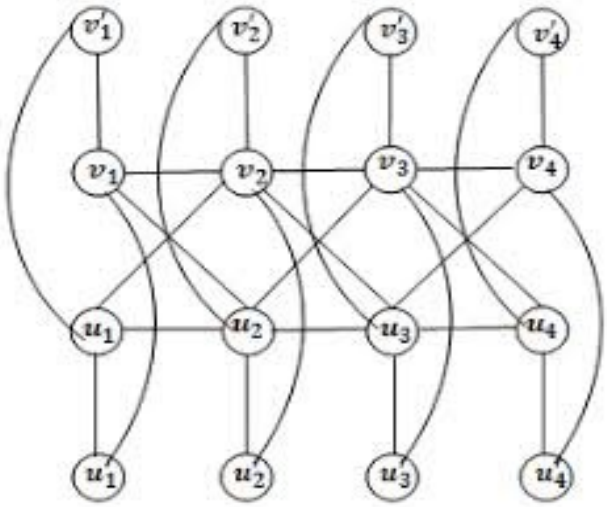
Thus the induced edge labels are distinct. Hence the above defined function provides mean labeling for split graph of comb.

Example:2

Mean labeling for split graph of comb



Theorem:**3** $D2(comb)$ admits mean labeling



Proof: Consider two copies of comb $G1$ and $G2$. Let $\{vi, 1 \le i \le n\}$ and $\{vi\,', 1 \le i \le n\}$ be the vertices of comb $G1$ Let $\{ui, 1 \le i \le n\}$ and $\{ui\,', 1 \le i \le n\}$ be the vertices of comb $G2$ The ordinary labeling for $D2(comb)$ as Let G be the shadow graph of comb

Define $f:(G) \to \{0,1,2,\ldots,q\}$ by

$f\ vi = 8i - 8\ if\ i\ is\ odd, 1 \le i \le n$

$= 8i - 7\ if\ i\ is\ even, 1 \le i \le n$

$f\ vi\,' = 8i - 7\ if\ i\ is\ odd, 1 \le i \le n$

$= 8i - 8\ if\ i\ is\ even, 1 \le i \le n$

$f\ ui = 8i - 4\ if\ i\ is\ odd, 1 \le i \le n$

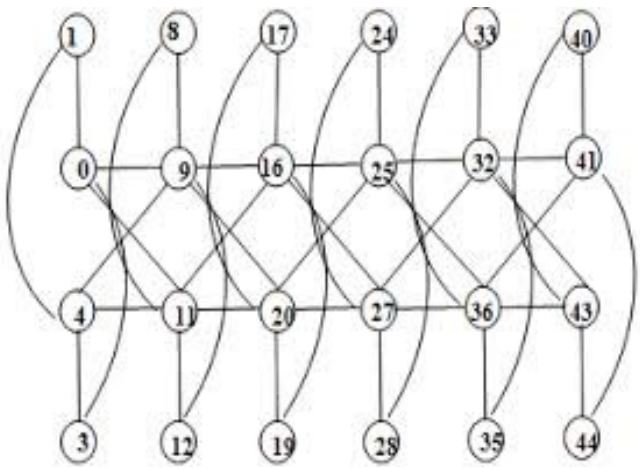
$= 8i - 5\ if\ i\ is\ even, 1 \le i \le n$

$f\ ui\,' = 8i - 5\ if\ i\ is\ odd, 1 \le i \le n$

$= 8i - 4\ if\ i\ is\ even, 1 \le i \le n$

Hence the above function provides mean labeling for shadow graph of comb.

Example 3:



Mean labeling for D2(comb)

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PRIME HARMONIOUS LABELLING OF SOME NEW GRAPHS

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ABSTRACT: *In this paper we have introduced prime harmonious labelling of some new graphs. Here we will discuss about some harmonious labelling techniques and some important theorems and examples based on those theorems for the wheel graph Wn, friendship graph Fn.*

Keywords: *Prime labelling, Harmonious labelling, Prime harmonious labelling, Wheel graph Wn, friendship graph Fn.*

1 Introduction:

All graphs in this paper are finite, simple and undirected. A graph is an ordered pair G=(V,E) comprising a set V of vertices or nodes or points together with a set E of edges or arcs or lines, which are 2-elements subsets of V.

2 Preliminaries:

2.1 Labelling of a Graph:

If the vertices of the graph are assigned values subject to certain conditions than it is known as graph labelling.

Most of the graph labelling problem will have following three common characteristics

- A set of numbers from which vertex labels are chosen,
- A rule that assigns a value to each edge,
- A condition that these values must satisfy.

2.2 Prime labelling:

Let G = G (V,E) be a graph. A bijection f : V [redacted] {1,2,3…….,|v|} is called prime labelling if for each edge e = {u, v} belong to E, we have GCD (f(u),f(v)) = 1. A graph which admits a prime labelling is called a prime graph.

2.3 Harmonious labelling:

Let G be a graph with q edges. A function f is called harmonious labelling of graph G if f : V→{0,1,2,3…….,q-1} is injective and the induced function f^* : E(G) [redacted] {0,1,2,….q} defined as f^* (e = uv) =(f(u) + f(v))(mod q) is bijective. A graph which admits a harmonious labelling is called a harmonious graph.

2.4 Prime harmonious labelling:

Let G be a graph with q edges. A function f is called prime harmonious labeling of graph G if f : V[redacted] {0,1,2,3…….,2q-1} is injective and the induced function f^* : E(G) [redacted] {0,1,2,….2q} is defined by

f^* (e =uv) = [redacted]

2.5 Wheel graph:

Wheel graph is a graph formed by connecting a single vertex to all vertices of a cycle (K1+ Cn).

2.6 Friendship graph:

A friendship graph Fn is a graph which consists of n triangles with a common vertex.

2.7 Graceful labelling:

A Graceful labelling of a graph with m edges is a labelling of its vertices with some subset of the integers between 0 and m inclusive such that no two vertices share a label, and such that each edge is uniquely identified by the positive, or absolute difference between its endpoints.

2.8 Edge graceful labelling:

A Labelling for simple graphs in which no two distinct edges connect the same two distinct vertices, no edge connects a vertex to itself, and the graph is connected.

2.9 Graph colouring:

A graph colouring is a subclass of graph labellings. A vertex colouring assigns different labels to adjacent vertices; an edge colouring assigns different labels to adjacent edges.

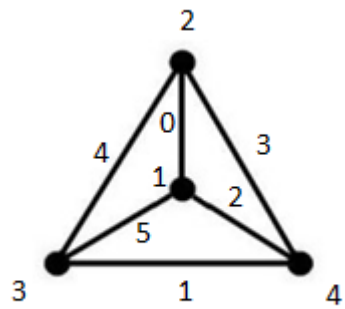
3 Prime harmonious labelling of graphs:

Theorem 3.1:

The wheel Wn is a prime harmonious labelling.

Proof:

Let u be the center vertex of the wheel Wn.



Let V1,V2,…….,Vn be the vertices of the leaves of wheel Wn.

Label the center vertex with 1and path vertices are labelled by 2 to n. It contains n+1vertex and 2n-1 edges.

(i.e) Wn= K_1+Cn

Joining all vertices of path P_n to the center vertex u.It satisfies the conditions of P_H .

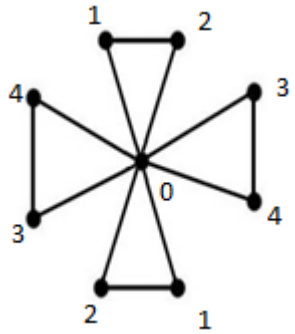
Therefore the wheel Wn is a prime harmonious labelling.

Theorem 3.2:

The friendship Fn is a prime harmonious labelling.

Proof:

Let V(Fn)={V1, V2,…….., V2n + 1} with V1 as the center vertex.



E(Tn) = {V1, V2, …….., V2n + 1} U{V2i, V2i + 1/1≤i≤n}

Define harmonious labelling:

F: V(Fn)→{1,2,3…….,V2n + 1}

E: E(Fn)→{1,2,3…….,3n} as follows

F(Vi)=I for 1≤i≤2n+1

Clearly, Fn is a prime harmonious labelling.

4 Applications of graph labelling:

The coding theory:

The design of certain important classes of good non periodic codes for pulse radar and missile guidance is equivalent to labeling the complete graph in such a way that all the edge labels are distinct. The node labels then determine the time positions at which pulses are transmitted.

The x-ray crystallography:

X-ray diffraction is one of the most powerful techniques for characterizing the structural properties of crystalline solids, in which a beam of X-rays strikes a crystal and diffracts into many specific directions. In some cases more than one structure has the same diffraction information. This problem is mathematically equivalent to determining all labeling of the appropriate graphs which produce a pre specified set of edge labels.

The communications network addressing:

A communication network is composed of nodes, each of which has computing power and can transmit and receive messages over communication links, wireless or cabled. The basic network topologies are include fully connected, mesh, star, ring, tree, bus. A single network may consist of several interconnected subnets of different topologies.

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RECYCLED DRAINAGE WATER FOR AGRICULTURAL PURPOSE BY USING GRAPH THEORY

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ABSTRACT: *One of the significant areas in mathematics is graph theory, which are second-hand in structural models his physical arrangements of various objects or technologies lead to new inventions and alterations in the existing environment for enhancement in those fields.We proposed the concerns of mathematical analysis in drainage water of India. Perspective is the development and implementation of particular computer control systems based on mathematical demonstrating of the procedures in the elements of drainage systems attractive into account changes in the time of their lively relationship. The aim of the work is to identify some graph theoretical models of drainage networks in large cities in terms of administration of employed modes for suitable sewer systems.*

Keywords: *Converse Root directed tree Model, Directed Path Model, Directed Star Model, Directed Ring Model.*

I. INTRODUCTION

Drainage is the natural or artificial removal of a surface's water and sub-surface water from an area to another area. The internal drainage of most agricultural soils is good enough to prevent severe water classification, but many soils and plants need some artificial drainage to improve production in agriculture and to manage water supplies.Drainage is an significant aspect of physical infrastructure and constitutes removal and disposal of surplus rain/irrigation water from the land. It has two aspects namely flood fortification and removal of storm water.

The proper functioning of metropolitan drainage (sewer) structure is one of the main aspects that determine the quality of environmental and sanitary settings of the settlement. For unswerving operation of sewer systems, it is necessary to implement active monitoring and management tools. The difficulties of monitoring and managing drainage systems of major cities in real time are complex and consistent problems. Now they repetition the most common method of process of sewer systems is manual control. With manual control, remedial actions for each well-ordered network structure depend on separate experience and the operator's level of training. Thus, administration is experiential rather than systematic, theoretically grounded. In large cities due to the high difficulty of metropolitan sewer systems, it is virtually impossible to safeguard for effective control and administration of their work manually. Perspective is the development and implementation of particular computer control systems based on mathematical demonstrating of the procedures in the elements of drainage systems attractive into account changes in the time of their lively relationship. The aim of the work is to identify some graph theoretical models of drainage networks in large cities in terms of administration of employed modes for suitable sewer systems.

Marshland soils may vital drainage to be used for the purpose of agriculture. In the northern United States and Europe, glaciations created numerous small lakes that gradually filled with humus to make marshes. Some of these were exhausted, using open ditches and trenches to make muck lands, which are predominantly recycled for high value crops such as vegetables.Using bipartite graphs, geodetic number for the distribution of waters from the Drainage and using fluid dynamics for drainage water flow.

Literature review:-

Over the years considerable quantity of literature have come up that deal either with Indian federalism. The available literature on Indian federalism focuses on hydrological, ecological and resettlement and rehabilitation aspect. In the same way works on federalism deals with in the setting of inter-state relations, power sharing and coalition politics. Attempt is made to compare and contrast concepts of Indian federalism and their associations in the context of Supreme Court judgement.

Objective:-

India is a quasi-federal state. In the Indian Constitution, water is in the State List while inter-state river is in the Union List. The Supreme Court of India has focused the central government to implement the RIL

project. There are broadly four issues like resettlement and rehabilitation of displaced, environmental concern, sharing of costs and benefits among states and international water conflict in the execution of RIL project. The paper attempts to inspect concepts of RIL and Indian federalism and their associations in the context of Supreme Court judgement.

II. Research methods:-

2.1Graph theory in Irrigation

The water supply to an irrigation system is directly related to agricultural productivity. An important factor that influences the effectiveness of an irrigation system is the design of a canal network that meets the water requirement of disparate locations. The amount of water conveyed down a canal is limited by its capacity. Canal capacity in diverse areas depends on various factors, such as land structure and topography (Johnson 2008), as well as its morphology.

Considered here is a stationary permanent continuous-flow irrigation system where a canal water-distribution system has been determined, which receives water from a main source and distributes the maximum possible amount of water per unit of time to discrete locations at the minimum possible construction cost. This is after the canal's capacity limitations have been taken into account.

Graph theory (Harary 1969) is used to deal with the problem, namely that an irrigation canal network N can be expressed by a weighted connected acyclic directed graph *G= (V, E, C)* where the nodes of V=$(v_0, v_1, v_2, \dots v_n)$ represent the water demand locations that may express interfarm distributors that deliver water to farms or crop-rotation sites. The arcs of E=[$(v_i, v_j), v_i, v_j \in V$] denote the potential canals to be constructed. The element of C=[$c(v_i, v_j)$] are the capacity of canal $(v_i, v_j)\in$ E. In the case where there are existing canals or pipelines, the construction cost of the corresponding arcs is considered to be equal to zero.

A simple path P_{ij} of N is an ordered sequenc $P_{ij} = (v_i, v_{i+1}, v_{i+2}, \dots v_{i+k} = v_j)$ such that $(v_i, v_{i+1})\in$ E; *i = 1,2,.....k-1* and where every node appears only once. Here, only simple path will be considered. For any $(v_i, v_j)\in$ E,v_i is a predecessor of v_i, and v_j is a successor of v_i. The predecessors and successors of $v_i\in$ V are the elements of sets $\Gamma^-(v_i)$and $\Gamma^+(v_i)$, respectively. The procedures devised in the following sections are expressed in a Fortran-like pseudocode language.

2.1. Structured arrangements of drainage systems

Sewer network are plan according to absorbed gravity notion, which delivers that waste water mostly pass through the sewage network under the effect of gravity. When gravity flow of sewage is impossible, sewage Treatment Plant (STP) are set, which propel liquid in the above-located areas. Usually arrangements of sewer networks are signifies in the form of directed, coherent graph where the sources are presented as vertices and drainage pipes and gatherers - in the form of edges. Depending on the specific planning and altitude structures of the city, sewer network structure presented in the following methods

Converse Root directed tree Model (CRDM): this method is the most common and can be used for an area that has one central sewage treatment plant (STP), so all sewage water enter these STP.

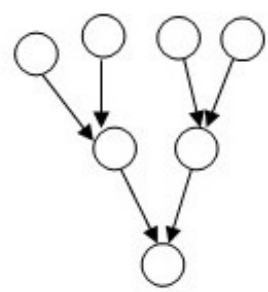

Figure 2.1: Convex tree Model

Directed Path Model (DPM): this method is used for big area that has one central sewage treatment plant (STP), so all sewage water enter these STP.

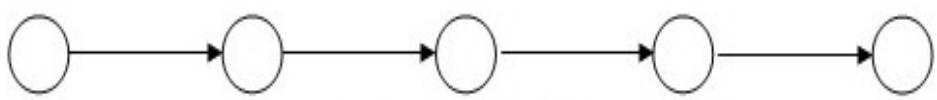

Figure 2.2: Directed Path Model

Directed Acyclic Model (DAM): this model is used for large areas with two or more central STP with presence of some connectivity of networks.

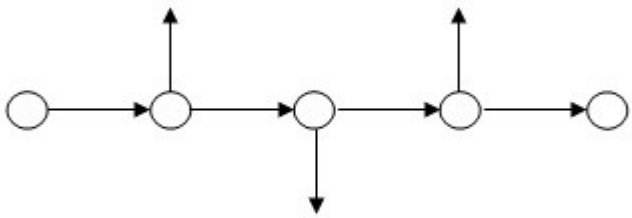

Figure 2.3: Directed Acyclic Model

Directed Star Model (DSM): this model is used for big area that has one central sewage treatment plant (STP), so all sewage water enter these STP.

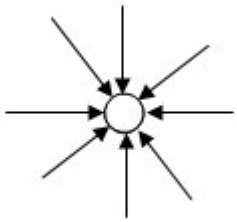

Figure 2.4: Directed Star Model

Directed Ring Model (DRM): this model is used for big area that has one or Two central sewage treatment plant (STP), so all sewage water enter these STP and extract clean water from other source.

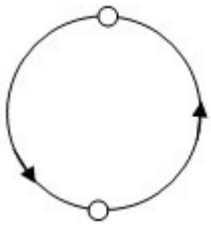

Figure 2.5: Directed Ring Model

Directed Gird Model (DGM): this model describes the sewer system, with collecting of all its areas in and centralized in one sewage treatment plant (STP)

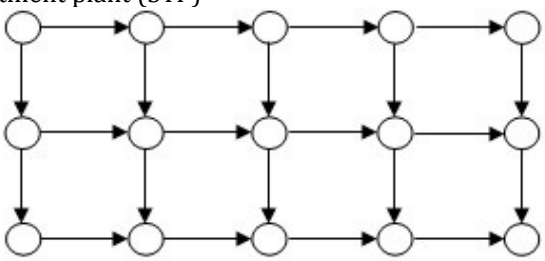

Figure 2.6: Directed Gird Model

IV.Conclusions

The method presented in the preceding paragraphs is a heuristic procedure that can in a feasible execution time solve situations concerning large networks, namely with hundreds demand locations since the time complexity of procedure BTR is a low polynomial function O(n^2), where n is the number of demand locations.

The method presented here can be effectively applied to a pipeline drainage system, as well, since the cost and the capacity of the pipes available on the market are known (Irrigation Pipe Price 2016).

Real-life irrigation systems are affected by a diverse number of components, such as the water-flow velocity, amount of water required at certain locations, etc. In certain cases, the framework of the BTR procedure can be applied to deal with these factors. A bottleneck tree in an irrigation canal network comprises the

minimum number of canals necessary in order to connect all the demand locations with the water source. Obviously, various bottleneck trees can be designed; however, in this paper a new method is proposed that determines the bottleneck tree with the minimum possible canal construction cost. The canals of an irrigation or pipeline drainage system are arranged in such a way so that at the same time unit they provide the maximum possible supply to the demand locations.

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On Soft Slightly *rgb* -Continuous Functions

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ABSTRACT: *In this paper, we introduce and study the concept of soft slightly rgb - continuous functions which is weaker than soft rgb -continuous functions and obtain its fundamental properties. The relationship between soft slightly rgb -continuity and other related functions is also analyzed.*

Keywords: *Soft rgb -closed set; soft rgb -open set; soft clopen set; soft rgb -continuity; soft slightly gb -continuity; soft slightly rgb -continuity.*

1 Introduction

Molodtsov [10] initiated the concept of soft set theory as a new mathematical tool and presented the fundamental results of the soft sets. Muhammad Shabir and Munazza Naz [11] introduced soft topological spaces and the notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation aioms. Kharal et al. [9] introduced soft function over classes of soft sets. Cigdem Gunduz Aras et al., [4] in 2013 studied and discussed the properties of Soft continuous mappings which are defined over an initial universe set with a fixed set of parameters. In this paper soft slightly rgb -continuous functions has been defined and proved that it includes the class of soft slightly gb -continuous and soft rgb -continuous functions. We investigated the properties of soft slightly rgb -continuous functions in terms of composition and restriction. The behavior of soft rgb -compactness and soft rgb -connectedness is studied under soft slightly rgb -continuous functions.

2. Preliminaries

We recall the following definitions, which will be used throughout this dissertation.

Deftnition 2.1. [10] Let X be an initial universal set and E be the set of parameters. Let P (X) denote the power set of X and $A \subseteq\tilde{} E$. A pair (F, A) is called a soft set over X , where F is a mapping given by $F : A \to P(X)$.

The collection of soft sets (F, A) over a universe X and the parameter set A is a family of soft sets denoted by S(X) .

Deftnition 2.2. [6] Let (X, τ, E) be a soft topological space. A soft set (F, E) is called soft regular closed set, if Cl(int(F, E)) = (F, E) . The complement of soft regular closed set is a soft regular open set.

Deftnition 2.3. [1] Let (X, τ, E) be a soft topological space. A soft set (F, E) is called a soft generalized closed set (briefly soft g -closed), if $Cl(F, E) \subseteq\tilde{} (U, E)$, whenever $(F, E) \subseteq\tilde{} (U, E)$ and (U, E) is soft open set in X . The complement of soft g -closed set is a soft g -open set.

Deftnition 2.4. [1] Let (X, τ, E) be a soft topological space. A soft set (F, E) is called a soft b -closed set, if $Cl(int(F, E)) \cap\tilde{} int(Cl(F, E)) \subseteq\tilde{} (F, E)$. The complement of soft b -closed set is a soft b -open set.

Note 2.5. [2] The intersection of all soft b -closed sets of (X, E, τ) containing (F, E) is called a soft b -closure of (F, E) and is denoted by bCl(F, E) .

Deftnition 2.6. [2] Let (X, τ, E) be a soft topological space. A soft set (F, E) is called a soft generalized b -closed set (briefly soft gb -closed), if $bCl(F, E) \subseteq\tilde{} (U, E)$, whenever $(F, E) \subseteq\tilde{} (U, E)$ and (U, E) is soft open set in X . The complement of soft gb -closed set is a soft gb -open set.

Deftnition 2.7. [2] Let (X, τ, E) be a soft topological space. A soft set (F, E) is called a soft regular generalized b -closed set (briefly soft rgb -closed set), if $bCl(F, E) \subseteq\tilde{} (U, E)$, whenever $(F, E) \subseteq\tilde{} (U, E)$ and (U, E) is soft regular open set in X . The complement of soft rgb –closed set is a soft rgb -open set.

Deftnition 2.8. [5] Let (X, τ, E) and (Y, τ ′, E) be two soft topological spaces. A soft function f : (X, τ, E) → (Y, τ ′, E) is said to be a soft g -continuous (soft b -continuous, soft regular continuous), if f −1((F, E)) is soft g -closed set (soft b -closed, soft regular closed) in X , for every soft closed set (F, E) in Y .

Deftnition 2.9. [7] A soft set (F, E) is said to be a soft clopen set if (F, E) is both soft open set and soft closed set.

Deftnition 2.10. [6] Let (X, τ, E) and (Y, τ', E) be two soft topological spaces. A soft function $f : (X, \tau, E) \to (Y, \tau', E)$ is said to be soft contra rgb -continuous if $f^{-1}(F, E)$ is soft rgb -open set in X , for every soft closed set (F, E) in Y .
Deftnition 2.11. [3] A space (X, τ, E) is called a soft rgb- T 12 space, if every soft rgb -closed set is soft gb -closed set (or) equivalently every soft rgb -open set is soft gb -open set.
Deftnition 2.12. [8] A space (X, τ, E) is said to be soft rgb -compact, if every soft rgb -closed cover of X has a finite subcover.
Deftnition 2.13. [8] A space (X, τ, E) is said to be soft countably rgb -compact, if every soft rgb -closed countably cover of X has a finite subcover.
Deftnition 2.14. [3] A space (X, τ, E) is said to be soft rgb -connected provided that X cannot be written as the union of two disjoint non-empty soft rgb -closed sets.
Deftnition 2.15. [3] A soft topological space (X, τ, E) is called a soft rgb - T0 space, if for each pair of distinct soft point x and y in X , there exist soft rgb -open set (F, E) and (tt, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (tt, E)$ and $x \notin (tt, E)$.

3 Soft Slightly rgb -Continuous Functions

In this chapter, we introduce the concept of soft slightly rgb -continuous functions and basic properties of soft slightly rgb -continuous functions are investigated.
Deftnition 3.1. Let (X, τ, E) and (Y, τ', E) be two soft topological spaces. A soft function $f : (X, \tau, E) \to (Y, \tau', E)$ is said to be soft slightly gb -continuous if $f^{-1}((F, E))$ is soft gb -closed set in X for each soft clopen subset (F, E) in Y .
Deftnition 3.2. Let (X, τ, E) and (Y, τ', E) be two soft topological spaces. A soft function $f : (X, \tau, E) \to (Y, \tau', E)$ is said to be soft slightly rgb -continuous if $f^{-1}((F, E))$ is soft rgb - closed set in X for each soft clopen subset (F, E) in Y .
THEOREM 3.3. The following statements are equivalent for a function $f : (X, \tau, E) \to (Y, \tau', E)$

(i) f is soft slightly rgb -continuous.
(ii) For every soft clopen subset (F, E) of Y , $f^{-1}(F, E)$ is soft rgb -open set in X .
(iii) For every soft clopen subset (F, E) of Y , $f^{-1}(F, E)$ is soft rgb -clopen set in X .

Proof: (i) → (ii): Let (F, E) be soft clopen subset in Y . Then $Y \setminus (F, E)$ is soft clopen subset in Y . Since f is a soft slightly rgb -continuous function, $f^{-1}(Y \setminus (F, E))$ is soft rgb -closed set in X . $f^{-1}(Y \setminus (F, E)) = X \setminus f^{-1}(F, E)$ is soft rgb -closed set in X implies $f^{-1}(F, E)$ is soft rgb -closed set in X .
(ii) → (iii): Let (F, E) be soft clopen subset in Y . Then $Y \setminus (F, E)$ is soft clopen subset in Y . By (ii), $f^{-1}(Y \setminus (F, E))$ is soft rgb -open set in X . Hence $f^{-1}(F, E)$ is soft rgb -closed set in X implies $f^{-1}(F, E)$ is soft rgb -clopen in X .
(iii) → (i): Let (F, E) be a soft clopen subset in Y . By (iii), $f^{-1}(F, E)$ is soft rgb -clopen subset in X . This implies $f^{-1}(F, E)$ is soft rgb -closed set in X . Hence f is soft slightly rgb -continuous.
THEOREM 3.4. Every soft slightly gb -continuous functions is soft slightly rgb -continuous functions.
Proof: Let $f : (X, \tau, E) \to (Y, \tau', E)$ be a soft slightly gb -continuous. Let (F, E) be any soft clopen subset in Y . Then $f^{-1}((F, E))$ is soft gb -closed set in X . Since every soft gb –closed set is soft rgb -closed set, $f^{-1}((F, E))$ is soft rgb -closed set in X . Therefore f is soft slightly rgb -continuous.
Remark 3.5. The converse of the above theorem is not true in general as shown in the following examples.
Example 3.6. Let $X = \{x1, x2\}$, $Y = \{y1, y2\}$ and $E = \{e1, e2\}$. Then

$\tau = \{\tilde{\varphi}, \tilde{X}, (F1, E), (F2, E), \ldots, (F8, E)\}$ is a soft topological space over X and $\tau' = \{\tilde{\varphi}, \tilde{Y}, (tt1, E), (tt2, E), \ldots, (tt6, E)\}$ is a soft topological space in Y , where $(F1, E), \ldots, (F8, E)$

and $(tt1, E), \ldots, (tt6, E)$ are soft sets over X and Y , defined as follows :

$F1(e1) = \{x1\}, F1(e2) = \{x2\}$ $\quad$ $F2(e1) = \{x2\}, F2(e2) = \{x2\}$

$F3(e1) = \{x1\}, F3(e2) = \{x1\}$ $\quad$ $F4(e1) = \{x1, x2\}, F4(e2) = \{x1, x2\}$

$F5(e1) = \{x1, x2\}, F5(e2) = \{x1\}$ $\quad$ $F6(e1) = \varphi, F6(e2) = \varphi$

F7(e1) = {x1, x2}, F7(e2) = {x2} F8(e1) = {x1, x2}, F8(e2) = φ

tt1(e1) = {y1}, tt1(e2) = {y2} tt2(e1) = {y1}, tt2(e2) = {y1}

tt3(e1) = {y2}, tt3(e2) = {y2} tt4(e1) = {y1, y2}, tt4(e2) = {y1, y2}

tt5(e1) = {y2}, tt5(e2) = {y1} tt6(e1) = {y1}, tt6(e2) = φ .

If we define the mapping f : (X, τ, E) → (Y, τ ′, E) is defined as u(x1) = y1 and u(x2) = y2 , then f is soft slightly rgb -continuous but not soft slightly gb -continuous, since f −1(tt3, E) = (F3, E) is not soft gb -closed set.

THEOREM 3.7. Let (X, τ, E) be a soft rgb - T1/2 space. Then the function f : (X, τ, E) → (Y, τ ′, E) is soft slightly rgb -continuous if and only if it is soft slightly gb -continuous.

Proof: Let (F, E) be soft clopen subset in Y . Since f is soft slightly rgb -continuous, f −1((F, E)) is soft rgb -closed set in X . Now f −1((F, E)) is soft gb -closed set in X as (X, τ, E) is soft rgb - T1/2 space. Therefore f is soft slightly gb -continuous. Conversely, Every soft slightly gb -continuous is soft slightly rgb -continuous.

THEOREM 3.8. Suppose S˜rg Bc(X) is soft gb -open set under arbitary unions. Let f : (X, τ, E) → (Y, τ ′, E) be a function. Then f is soft slightly rgb -continuous if and only if for each point x ∈ X and each soft clopen subset (tt, E) containing f (x) , there exists a soft rgb -closed set (F, E) containing x such that f (F, E) ⊆˜ (tt, E) .

Proof: Assume that f is soft slightly rgb -continuous. Let x ∈ X and (tt, E) be a soft clopen subset in Y containing f (x) . That is f (x) ∈ (tt, E) . Also f −1((tt, E)) is soft rgb -closed set in X . If we take (F, E) = f −1(tt, E) , then x ∈ (F, E) and f (F, E) ⊆˜ (tt, E) .

Conversely, let (tt, E) be a soft clopen subset of Y and let x ∈ f −1((tt, E)) . Since f (x) ∈ (tt, E) , there exists a soft rgb -closed set (Fx, E) in X containing x such that (Fx, E)⊆˜f −1((tt, E)) .

We obtain f −1((tt, E)) = S˜ {(Fx, E) : x ∈ f −1((tt, E))} . Thus f −1((tt, E)) is soft rgb -closed set in X . Hence f is soft slightly rgb -continuous.

Deftnition 3.9. A space (X, τ, E) is said to be soft locally indiscrete if every soft open set of X is soft closed set in X .

THEOREM 3.10. If a function f : (X, τ, E) → (Y, τ ′, E) is soft slightly rgb -continuous and (Y, τ ′, E) is soft locally indiscrete, then f is soft rgb -continuous.

Proof: Let (F, E) be a soft open set in Y . Since Y is soft locally indiscrete, every soft open set is soft closed set. Therefore (F, E) is soft closed set in Y . Since f is soft slightly rgb -continuous, f −1((F, E)) is soft rgb -closed set in X . Hence f is soft rgb -continuous.

THEOREM 3.11. If a function f : (X, τ, E) → (Y, τ ′, E) is soft slightly rgb -continuous and (X, τ, E) is soft rgb - T 12 space, then f is soft slightly gb -continuous.

Proof: Let (F, E) be a soft clopen set in Y . By hypothesis, f −1((F, E)) is soft rgb -closed set in X . Since X is soft rgb - T1/2 space, f −1((F, E)) is soft gb -closed set in X . Hence f is soft slightly gb -continuous.

THEOREM 3.12. Let f : (X, τ, E) → (Y, τ ′, E) and g : (Y, τ ′, E) → (Z, τ ∗, E) be functions. Then the following properties hold :

(i) If f is soft rgb -irresolute and g is soft slightly rgb -continuous, then gof : (X, τ, E) → (Z, τ ∗, E) is soft slightly rgb -continuous.
(ii) If f is soft rgb -irresolute and g is soft rgb -continuous, then gof : (X, τ, E) → (Z, τ ∗, E) is soft slightly rgb -continuous.
(iii) If f is soft rgb -irresolute and g is soft slightly gb -continuous, then gof : (X, τ, E) → (Z, τ ∗, E) is soft slightly rgb -continuous.
(iv) If f is soft rgb -irresolute and g is soft contra rgb -continuous, then gof : (X, τ, E) → (Z, τ ∗, E) is soft slightly rgb -continuous.

Proof: (i) Let (F, E) be a soft clopen subset in Z . Then g−1((F, E)) is soft rgb -closed set in Y . Since f is soft rgb -irresolute, f −1((g−1(F, E))) is soft rgb -closed set in X . Since g is soft slightly rgb -continuous, so (gof)−1((F, E)) = f −1((g−1(F, E))) , gof is soft slightly rgb - continuous.

(ii) Let (F, E) be a soft clopen subset in Z . Then g−1((F, E)) is soft rgb -closed set in Y . Since f is soft rgb - irresolute, f −1((g−1(F, E))) is soft rgb -closed set in X . Since g is soft rgb -continuous, so (gof)−1((F, E)) = f −1((g−1(F, E))) , gof is soft slightly rgb -continuous.

(iii) Let (F, E) be a soft clopen subset in Z . Then g−1((F, E)) is soft gb -closed set in Y . Since f is soft gb -irresolute, f −1((g−1(F, E))) is soft rgb -closed in X . Since g is soft slightly gb -continuous, so (gof)−1((F, E)) = f −1((g−1(F, E))) , gof is soft slightly rgb -continuous.

(iv) Let (F, E) be a soft clopen subset in Z . Then g−1((F, E)) is soft rgb -open set and soft rgb -closed set in Y , since g is soft contra rgb -continuous. Also f is soft rgb -irresolute, f −1((g−1(F, E))) is soft rgb -open set and soft rgb -closed set in X . Hence gof is soft slightly rgb -continuous.

THEOREM 3.13. Let f : (X, τ, E) → (Y, τ ′, E) and g : (Y, τ ′, E) → (Z, τ ∗, E) be functions. If f is soft rgb -open surjective and gof : (X, τ, E) → (Z, τ ∗, E) is soft slightly rgb -continuous then g is soft slightly rgb -continuous.
Proof: Let (F, E) be any soft clopen set in (Z, τ ∗, E) . Since gof is soft slightly rgb -continuous functions, (gof)−1(F, E)) = f −1(g−1(F, E)) is soft rgb -open set in X . Since f is soft rgb –open surjective, then f (f −1)(g−1(F, E)) = g−1(F, E) is soft rgb -open in Y . Hence g is soft slightly rgb -continuous.
THEOREM 3.14. Let f : (X, τ, E) → (Y, τ ′, E) be surjective, soft rgb -irresolute, soft rgb - closed set and let g : (Y, τ ′, E) → (Z, τ ∗, E) be a soft function. Then gof : (X, τ, E) → (Z, τ ∗, E) is soft slightly rgb -continuous if and only if g is soft slightly rgb -continuous.
Proof: Assume that gof : (X, τ, E) → (Z, τ ∗, E) is soft slightly rgb -continuous and let (F, E) be a soft clopen subset in Z . Then f −1(g−1(F, E)) is soft rgb -closed set in X . Since f is surjective, f (f −1(g−1(F, E))) = g−1(F, E) is soft rgb -closed set in Y . Hence g is soft slightly rgb -continuous. Conversely, let g be soft slightly rgb -continuous and (F, E) be a soft clopen subset in Z , then g−1(F, E) is soft rgb -closed set in Y . Since f is soft rgb -irresolute, f −1(g−1(F, E)) is soft rgb -closed set in X . Hence gof is soft slightly rgb -continuous.
THEOREM 3.15. Let f : (X, τ, E) → (Y, τ ′, E) is soft slightly rgb -continuous and A is soft closed subset of X . Then f |A : A → (Y, τ ′, E) is soft slightly rgb -continuous.
∗
Proof: Let (F, E) be a soft clopen subset in Y . we have (f |A)−1((F, E)) = f −1((F, E)) ∩ A . Since f is soft slightly rgb -continuous and A is soft closed set of X . (f |A)−1((F, E)) is soft rgb -closed set in the relative topology of A . Thus f |A ia soft slightly rgb -continuous.

4. Soft rgb –Compact Space And Soft rgb Connected Space

Deftnition 4.1. A space (X, τ, E) is said to be:

(i) Soft mildly compact, if every clopen cover of X has a finite subcover.
(ii) Soft mildly countably compact, if every soft clopen countably cover of X has a finite subcover.
(iii) Soft mildly Lindelof, if every soft clopen cover of X has a countable subcover.
(iv) Soft rgb -Lindelof, if every soft rgb -closed cover of X has a countable subcover.
(v) Soft rgb -closed compact, if every soft rgb -closed cover of X has a finite subcover.
(vi) Soft countably rgb-closed compact, if every countable cover of X by soft rgb -closed sets has a finite subcover.

THEOREM 4.2. Let f : (X, τ, E) → (Y, τ ′, E) be a soft slightly rgb -continuous surjection. Then the following statements hold :

(i) If X is soft rgb -Lindelof, then Y is soft mildly Lindelof.
(ii) If X is soft countably rgb -compact, then Y is soft mildly countably compact.

Proof: (i) Let {(Fα, E) : α ∈ I} be any soft clopen cover of Y . Since f is soft slightly rgb - continuous, then we have {f −1(Fα, E) : α ∈ I} be a soft rgb -closed cover of X . Since X is soft rgb -Lindelof, there exists a countable subset I0 of I such that X = S{f −1(Fα, E) : α ∈ I} . Thus Y = S{(Fα, E) : α ∈ I0} and Y is soft mildly Lindelof.
(ii) Let {(Fα, E) : α ∈ I} be any soft clopen cover of Y . Since f is soft slightly rgb -continuous, then we have {f −1(Fα, E) : α ∈ I} be a soft rgb -closed countable cover of X . Since X is soft countably rgb -Lindelof there exists a finite subcover of {f −1(Fα, E) : α ∈ I} such that
i=1
X = Sn f =1
(Fi, E) . Thus Y = Sn (Fi, E) and hence Y is soft mildly countable compact.
THEOREM 4.3. Let f : (X, τ, E) → (Y, τ ′, E) be any soft slightly rgb -continuous surjection. Then the following statements hold:

(i) If X is soft rgb -closed compact, then Y is soft mildly compact.
(ii) If X is soft countably rgb -closed compact, then Y soft mildly countably compact.

Proof: (i) Let {(Fα, E) : α ∈ I} be any soft clopen cover of Y . Since f is soft slightly rgb - continuous, {f −1(Fα, E) : α ∈ I} is a soft rgb -closed cover of X . Since X is soft rgb –closed compact, there exists a finite subcover of {f i=1 =1
(Fα, E) : α ∈ I} such that X = Sn f −1(Fi, E) .
Thus Y = Sn (Fi, E) and hence Y is soft mildly compact.
(ii) Let {(Fα, E) : α ∈ I} be any soft countable clopen cover of Y . Since f is soft slightly rgb -continuous, {f −1(Fα, E) : α ∈ I} is a soft countable rgb -closed cover of X . Since X is soft countably rgb -closed compact, there exists a finite subcover of {f −1(Fα, E) : α ∈ I} such that i=1 X = Sn {f −1(F , E) . Thus Y = Si=1(F , E) and hence Y is soft mildly countably compact.
Corollary 4.4. If f : (X, τ, E) → (Y, τ ′, E) is soft slightly rgb -continuous and surjection and X is soft rgb -compact, then Y is soft mildly compact.
Proof: Let {(Fα, E) : α ∈ I} be any soft clopen cover of Y . Since f is soft slightly rgb - continuous. {f −1(Fα, E) : α ∈ I} be a soft rgb -closed cover of X . so there is a finite subset I0 of I such that X = S f −1(Fα, E) . Then Y = S (Fα, E) . Since f is soft surjection. Thus Y is soft mildly compact.
THEOREM 4.5. If f : (X, τ, E) → (Y, τ ′, E) is a soft slightly rgb -continuous and soft surjective function and X is soft rgb -connected, then Y is soft connected.
Proof: Suppose Y is not soft connected. Then there exist non-empty disjoint soft clopen (F, E) and (tt, E) of Y such that Y = (F, E) ∪˜ (tt, E) . Since f is soft slightly rgb continnuous. f −1(F, E) and f −1(tt, E) are non-empty soft rgb -closed in X .
Thus X is not soft rgb connected which is a contradiction. Hence Y is soft connected.
THEOREM 4.6. If f is a soft slightly rgb -continuous function from a soft rgb -connected space X onto any space Y , then Y is not a soft discrete space.
Proof: Suppose that Y is soft discrete. Let (F, E) be a proper non-empty soft closed subset of Y . Then f −1(F, E) is any proper non-empty soft rgb -clopen subset of X , which is a contradiction to the assumption that X is soft rgb -connected. Therefore Y is not a soft discrete space.
THEOREM 4.7. If a function f : X → ΠYα is soft slightly rgb -continuous, then Pαof : X → Yα is soft slightly rgb -continuous for each α ∈ I , where Pα is the projection of ΠYα onto Yα .
Proof: Let (Fα, E) be any soft clopen subset in Yα . Then P −1(Fi, E) is soft clopen in ΠYα . Hence (Pαof)−1(Fα, E) = (f −1(P −1(Fα, E))) is soft rgb -closed set in X . Therefore Pαof is soft slightly rgb -continnuous.

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A COMPARATIVE ANALYSIS OF COST MINIMIZING TRANSPORTATION PROBLEM BY ZERO POINT METHOD AND OTHER TRANSPORTATION METHODS

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ABSTRACT: *In real world situations, different sources and destinations impose different restrictions on supply and demand respectively according to their availability and requirement because of the changing economic and environmental conditions. It is also important that the cost required for transportation should be minimum. The problem of minimizing the total transportation has been studied since long and is well known. The objective of this problem is minimizing the transportation time and cost of goods which supply from one source to another source. In this paper we are comparing zero point method and other transportation method to solving cost minimizing transportation problem.*

Keywords: *Transportation problem, Zero point method, NWC method, VAM method, Optimum solution*

1 Introduction:

The transportation problem is a special category of Linear Programming Problem. It has been widely studied in Logistics and Operations Management where distribution of goods and commodities from sources to destinations is an important issue. The task of distributor's decisions can be optimized by reformulating the Distribution Problem as generalization of the classical Transportation Problem. The conventional Transportation Problem can be represented as a mathematical structure which comprises an Objective Function subject to certain Constraints. In classical approach, transporting costs from M sources or wholesalers to N destinations or consumers are to be minimized. The time minimizing transportation problem is a
special case of a transportation problem in which a time associated with each shipping route. Rather than minimizing time, the objective is to minimize the maximum cost to transport all supply to the destinations. The problem of minimizing the total transportation has been studied since long and is well known. In a Cost transporting goods is minimized to satisfy certain condition in respect of availabilities at sources and requirement at destinations.

2 Algorithm:

The Zero Point Method proceeds as follows:

Step 1. Construct the transportation table for the given transportation problem and then, convert into a balanced one if not.

Step 2. Subtract each row entries of the transportation table from the row minimum

Step 3. Subtract each column entries of the transportation table after using Step 2 from the column minimum.

Step 4. Check if each column demand is less then to the sum of the supplies whose reduced costs in that column are zero. Also check if each row supply is less than to sum of the column demands whose reduced costs in that row are zero. If so, go to Step 7 (Such reduced table is called the allotment table). If not go to Step 5

Step 5. Draw the minimum number of horizontal lines and verticals line to cover all the Zeros of the reduced transportation table such that some entries of rows(s) or / column(s) which do not satisfy the condition of the step 4, are not covered.

Step 6. Develop the new revised reduced transportation table as follows:

(i) Find the smallest entry of the reduced cost matrices not covered by any lines.

(ii) Subtract this entry from all the uncovered entries and the same to all entries lying at the intersection of any two lines, and then, go to step 4.

Step 7. Select a cell in the α – row or / and β- column of the reduced transportation table which is the only cell whose reduced time is zero and then allot the maximum possible to that cell. If such a cell does not occur for the maximum value, find the next maximum so that such a cell occurs. If such cell does not occur for any value, we select any cell in the reduced transportation table whose reduced time is zero.
Step 8. Reform the reduced transportation table after deleting the fully used supply point and the received demand points and also, modify it to include the not fully used supply point and not received demand points.
Step 9. Repeat Step 7 to Step 9 until all supply points are used and all demand points are fully received.
Step 10. This allotment yields a solution to the time transportation problem.

3 Numerical Example:
In this section we have taken a transportation problem to reduce the transportation Cost for essential commodity supply from one source to another source to determine an optimal solution so as to minimize the transportation cost and comparing methods between zero point methods to other methods.
Table 3.1

	W1	W2	W3	W4	Supply
F1	1	10	12	11	23
F2	15	21	22	9	41
F3	22	16	32	15	48
Demand	25	16	30	41	

Now, Σ ai = Σ bj = 112, the given transportation problem is balanced.

Zero point method: In this section we are applying zero point method to solve the problem:
Table 3.2

	W1	W2	W3	W4	Supply
F1	**23** 1	10	12	11	23
F2	**2** 15	21	**30** 22	9 9	41
F3	22	**16** 16	32	**32** 15	48
Demand	25	16	30	41	

We get the optimal cost for the transportation problem as 1650.

4 Regular methods:
In this section we are using regular method to solve cost minimization problem to get optimum cost, here we are using North West Corner Method (NWCR) and Vogel Approximation Method (VAM) to get feasible solution. In this section we solve NWCR,VAM.
4.1 North West Corner Method:

	W1	W2	W3	W4	Supply
F1	**23** 1	10	12	11	23
F2	**2** 15	16 21	**23** 22	9	41
F3	22	16	**7** 32	**41** 15	48
Demand	25	16	30	41	

We get the optimal cost for transportation problem as 1734.

4.2 Vogel Approximation Method:

	W1	W2	W3	W4	Supply
F1	1	10	**23** 12	11	23
F2	**25** 15	**9** 21	**7** 22	9	41
F3	22	**7** 16	32	**41** 15	48
Demand	25	16	30	41	

We get the optimal cost for transportation problem as 1721.

5 Conclusion:

The zero point method is a systematic procedure to solve the all type transportation problem because its easy to apply and utilized for all types of transportation problem. This an important tool for the decision makers when they are handling various types of logistic problems because other methods gives an optimal solution but zero point method gives optimal solution without help of any other modified method.

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A STUDY ON MATCHING GRAPHS

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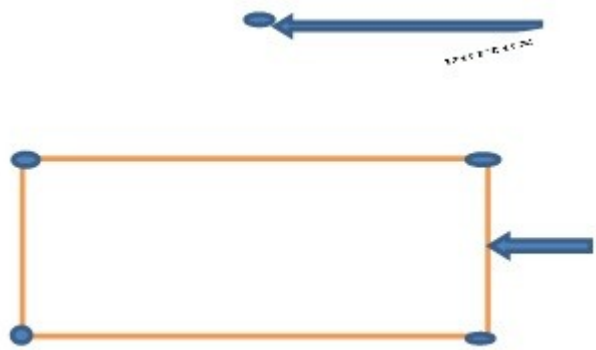
ABSTRACT: *This paper assumes basic knowledge of definitions and concepts as they pertain to graph theory. With that in mind, let's begin with the main topic of these notes: matching. For now we will start with general definitions of matching. Later we will look at matching in bipartite graphs then Hall's Marriage Theorem.*

Keywords:

INTRODUCTION

We start the review with some of basic definitions most of which are illustrated in fig1 Recall the degree d(v) of a vertex v is number of its neighbours so in the example d(s)= 3

If the edges of a graph are ordered pairs rather than unordered pairs one obtains a Directed graph



1.1 Definitions.

Definition 1.1.

A matching of graph G is a subgraph of G such that every edge shares no vertex with any other edge. That is, each vertex in matching M has degree one.

Definition 1.2.

The size of a matching is the number of edges in that matching.

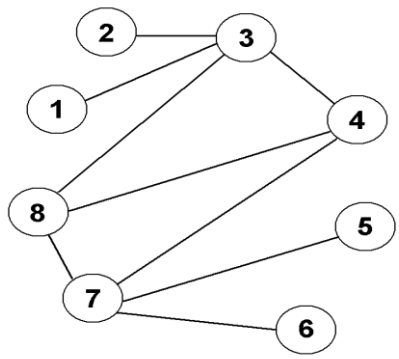


Figure 1

Consider the graph in Figure 1. Denote the edge that connects vertices i and j as (i; j). Note that f(3; 8)g is a matching. Obviously we can get more. The pairsf(3; 8); (4; 7)g also make a matching. That is a matching of size two. Can we geta matching of size three? Yup, it's f(2; 3); (4; 8); (5; 7)g. Can we do even better? Well, a

matching of size four means that every vertex is paired, but vertices 1 & 2must both be paired with vertex 3. So no, three is the best we can do. We call ita maximum matching.

Definition 1.3.
A matching is maximum when it has the largest possible size. Note that for a given graph G, there may be several maximum matchings.
Definition 1.4.
The matching number of a graph is the size of a maximum matching of that graph. Thus the matching number of the graph in Figurei 1 is three.
Definition 1.5.
A matching of a graph G is complete if it contains all of G's vertices. Sometimes this is also called a perfect matching. Thus no complete matching exists for Figure 1. 1.2. Matching in Bipartite Graphs.
Definition 1.6.
A bipartite graph is a graph whose vertices can be divided into two disjoint sets such that no edge connects two vertices of the same set. It is common to use the terms left and right to describe the two sets of vertices. A balanced bipartite graph is one that has an equal number of left and right vertices.
Definition 1.7.
Consider a subset S _ L of left vertices of bipartite graph G. Let n be the number of right vertices the set S is connected to. Then the deficiency D of set S is defined as:

$$D(S) = \begin{cases} |s| - n & if\ positive \\ o & otherwise \end{cases}$$

Define the left deficiency DL of a bipartite graph as the maximum such D(S) taken from all possible subsets S. Right deficiency DR is similarly defined.
As an example, let's consider the complete bipartite graph K3;2. Recall that Km;n stands for a complete bipartite graph with m left vertices and n right vertices.

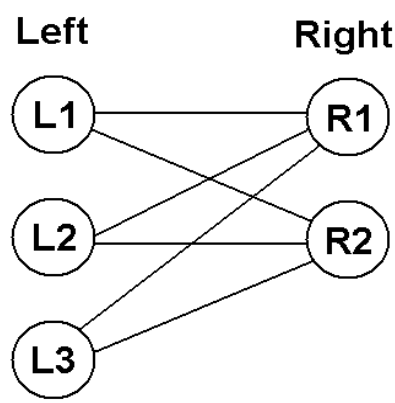


Figure 2. K3;2

If we consider the subset S = fL1;L2g, we see that the deficiency of this subset is zero, as the number of neighbours of S is at least the size of S. But if we consider the subset S = fL1;L2;L3g, we _find a deficiency of one, as these three left vertices connect to only two right vertices. Since there exists no larger deficiency, the left deficiency of K3;2 has a value of one. Now let's look at the right side of our graph. Consider the subset S = fR1g. Since this vertex connects to 3 left vertices, this subset has a deficiency of zero. In fact it is easy to see that the right decency of our graph is also zero. With these concepts in place, we will now over a proposition, to be proven later.
1.3. Hall's Marriage Theorem. Philip Hall in 1935 gave us the condition for when a complete matching is possible in a bipartite graph. An easy was to visualize this is to consider the following situation: Suppose we are pairing up N boys and N girls (if they were not both N then clearly there is no way for a matching of our bipartite graph to be complete). Now each girl comes up with a list of acceptable mates that she likes, some subset of the N boys. Since these boys are of the gentlemanly type, none of them will reject a proposal if given to them. This situation can be represented by a bipartite graph, where an edge represents the event that a specific girl likes a specific guy. One such possible arrangement is given in Figure 3.

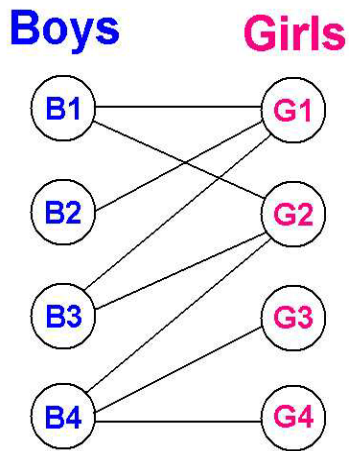


Figure 3

We can now state Hall's marriage condition:

Definition 1.9.

Hall's marriage condition holds when every subset of r girls likes at least r boys. This is exactly the same as saying Hall's marriage condition holds when DL(G) = 0 (or DR(G) = 0) for a balanced bipartite graph G. An equivalent condition can be created by interchanging `boy' and `girl' in the definition above. Also note that the marriage condition only applies to the case where we have an equal number of boys and girls, i.e. when we have a balanced bipartite graph. Can you see how you would relate this condition to a bipartite graph? Here is the main theorem of this section.

Theorem 1.10 (Hall's Marriage Theorem).

Hall's marriage condition is both necessary and sufficient for the existence of a complete match in a bipartite graph. That is to say, i_ Hall's marriage condition holds for a bipartite graph, then a complete matching exists for that graph.

Looking at Figure 3 we can see that this graph does not meet the marriage condition. If we take the set of girls fG3;G4g, they are both paired with only one boy, thus they have a deficiency of one. This violates the marriage condition. Here it is easy to see that no complete matching exists because of this: only at most one of fG3;G4g could get married, the other one won't be. If we look at the marriage condition from the boy's side, we also see a violation of the marriage condition. If we look at the set of boys fB1;B2;B3g, we see that they collectively like the same two girls. This set also has a deficiency of one. So a similar problem arises: at most two boys from our set can possibly get married; the third one is doomed to be a bachelor forever.

4 NOTES ON MATCHING

Notice that the marriage theorem is a subcase of Proposition 1.8, where N =|L| = |R| and DL(G) = 0. Both theorems say that in this case, the matching number is N, or in other words there exists a complete matching. It is easy to see that the marriage condition is necessary for a complete matching, but Hall's marriage theorem asserts that it is also sufficient. Proof of Hall's Marriage Theorem. Since necessity is easy to see, we need to prove that the marriage condition is also suficient. That is to say, if the marriage conditionholds, then there exists a complete matching.

We will use induction to prove our desired result. Given a balanced bipartite graph G, assume that the marriage condition holds. We will induct on r, the size of a subset S of left vertices of our graph G. What we need to do is show that if the marriage theorem holds for r, then it also holds for r + 1, and eventually the entire graph G. But first the base case, where r = 1. If r = 1 and the marriage condition holds true for any subset of size one, then clearly it can be paired with a right vertex. Then we have a complete matching for any one vertex. Base case is done.

Now for the inductive step. Assume that for any r-sized subset S, the marriage condition holds and so does the marriage theorem. Now we need to show, based on the previous assumptions, that the marriage theorem also holds true for r +1. Let the set of vertices that S connects to be denoted as S0. Now consider any (r +1)th left vertex, vr+1. Now we split the problem into three cases.

Case 1. The vertex vr+1 is connected to some vertex not in S0. Then clearly there exists a complete matching among these (r + 1) vertices. This is illustrated in Figure 4.

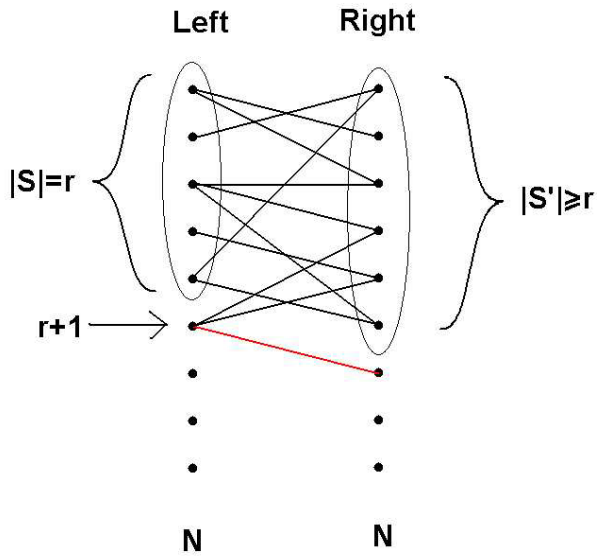


Figure 4. Case 1

create an (r + 1)-sized matching. So WLOG, let's remove vertex v1 and consider all the other vertices (including vertex vr+1) as our r-sized subset. Let's call this subset Sr. Now consider all complete matchings for Sr. Let r be the subset of S0 that is in every complete matching of Sr. If vertex v1 is connected to only points in S0 r, then this contradicts the marriage condition which we have assumed to be true. Thus v1 is connected to some point not in S0r, thus a complete matching exists for all r+!1 points. This is illustrated in Figure 5. The bold edges represent one possible complete matching for vertices v2; : : : ; vr+1. Because of our original assumptions, the red edge is guaranteed to exist, thus establishing our inductive step.

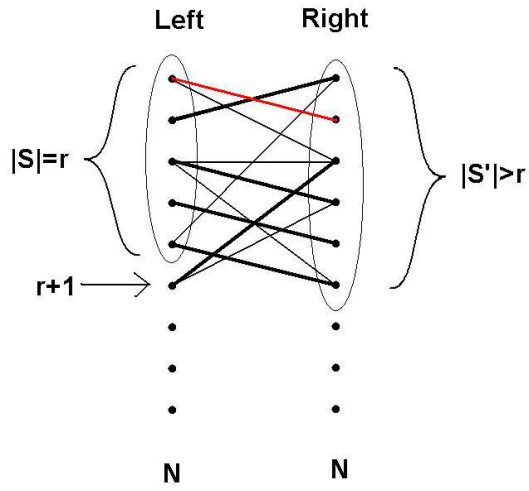


Figure 5. Case 3. The red edge is guaranteed to exist.

With the completion of our inductive step, it can be seen that the marriage theorem is true for our entire graph G. And since the size of G was not stipulated, the marriage theorem thus holds true for any such arbitrarily large graph.

Example 1.14. Take a regular deck of 52 playing cards and randomly deal them into 13 piles of 4 cards each. Then by the marriage theorem, there exists a way to pull out one card from each pile so that you have one card from every rank (ace, 2,: : :, king).

Proof.

To prove the example above, we must _rst represent the problem as a bipartite graph. Then we must show that the marriage condition holds for our graph. After we have done that, we can use Hall's Marriage Theorem to prove the example. So the firrst step is to model this problem as a bipartite graph. How? Have

each of the 13 piles represent a vertex on the left, and each of the 13 ranks represent avertex on the right. An edge exists between two vertices i_ that pile contains that rank. For example, if an ace is found in pile number 1, we draw an edge between the top two vertices. The keen observer will note the possibility of multiple edges. Multiple edges will arise when two cards of the same rank (say two kings) appearing the same pile. So this is how we model our problem as a bipartite graph. The next step is to show that the marriage condition holds in our graph. Notice that each vertex on the left (the piles) have degree 4 (because they have 4 cards in them), and every vertex on the right (the ranks of the cards) also have degree 4 (because each of the 4 cards in each rank must be placed in a pile). Now we have a specific case of one of your homework problems. With the proof of said homework problem, the marriage condition is shown to hold true, and by Hall's Marriage Theorem, there exists a complete matching in our bipartite graph. Applying this property back to our piles of cards, we see that it is possible to pull one of each rank from the 13 piles, according to the edges in our complete matching that we have shown must exist.

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