

Edge Trimagic Graceful Labeling of Some Disconnected Graphs

¹J. A. Jose Ezhil & ²M. Regees & ³T. Shyla Isac Mary

¹Researchscholar, Reg. No – 18113112092011, Department of Mathematics, Nesamony Memorial Christian College, Marthandam – 629165, kanyakumari District, Tamil Nadu, India.

²Assistant Professor, Department of Mathematics, Malankara Catholic College, Mariagiri, Kaliakavilai – 629153, Tamilnadu, India.

³Assistant Professor, Department of Mathematics, Nesamony Memorial Christian College, Marthandam – 629165, kanyakumari District, Tamil Nadu, India.

Received: February 11, 2019

Accepted: March 19, 2019

ABSTRACT: A (p, q) graph G is called an edge trimagic graceful if there exists a bijection $\varphi : V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, p+q\}$ such that for each edge xy in $E(G)$, $|\varphi(x) - \varphi(xy) + \varphi(y)| = c_1$ or c_2 or c_3 , where c_1, c_2 and c_3 are constants. In this paper, we proved some disconnected graphs like, $(C_m \boxtimes K) \boxtimes R, nP_3, K_{1,l} \cup K_{1,m} \cup K_{1,n}nC_4$ and $(C_m \odot K_1) \boxtimes C_n$ are edge trimagic graceful graphs.

Key Words: Graph, disconnected graph, trimagic, graceful, trimagic graceful.

AMS Subject Classification: 05C78

1. INTRODUCTION

Labeling of a graph G is an assignment of labels to either the vertices or the edges or both subject to certain conditions. Graph labeling was first introduced in 1960's. In 1970, Kotzig and Rosa[6] defined, an edge magic labeling of graph G is a bijection $f: V \cup E \rightarrow \{1, 2, \dots, p+q\}$ such that, for each edge $uv \in E(G)$, $f(u) + f(uv) + f(v)$ is a magic constant. A few years later, S. W. Golomb renamed β -labeling as graceful labeling as it is known today [4]. In 2004 J. Baskar Babujee introduced Edge Bimagic labeling [1]. Marimuthu and Balakrishnan [7] introduced the edge magic graceful labeling of a graph. In 2013, C. Jayasekaran, M. Regees and C. Davidraj [8] introduced the edge trimagic total labeling of graphs.

A graph is connected if every pair of points are joined by a path. A graph which is not connected is disconnected. Graphs G_1 and G_2 have disjoint vertex set V_1 and V_2 and the edge set E_1 and E_2 respectively. Their union $G = G_1 \cup G_2$ has, as expected, $V = V_1 \cup V_2$ and $E = E_1 \cup E_2$. A simple graph in which there exists an edge between every pair of vertices is called a complete graph and it is denoted by K_n . A bigraph G is a graph whose vertex set V can be partitioned into two subsets V_1 and V_2 such that every edge of G joins a point of V_1 to a point of V_2 . If G contains every edge joining V_1 and V_2 , then G is a complete bigraph. If V_1 and V_2 have m and n vertices, we write $G = K_{m,n}$. A star is a complete bigraph $K_{1,n}$. Further reference, we have used the survey on graph labeling by J. A. Gallian (2018) [5].

We can see some Super edge bimagic and trimagic labeling for some disconnected graphs are proved in [2, 3, 9]. In this paper we are going to prove some of the disconnected graphs such as $(C_m \odot K_1) \cup P_n, nP_3, K_{1,l} \cup K_{1,m} \cup K_{1,n}nC_4$ and $(C_m \odot K_1) \boxtimes C_n$ are edge trimagic graceful graphs.

2. EDGE TRIMAGIC GRACEFUL LABELING OF SOME DISCONNECTED GRAPHS

Theorem 2.1: The graph $(C_m \odot K_1) \cup P_n$ admits an edge trimagic graceful labeling for all values of m and n .

Proof: Let $w_1w_2\dots w_n$ be the path P_n . Let $u_1u_2\dots u_mu_{m+1}$ be the cycle C_m and let v_i be the vertex which is joined to the vertex u_i of the cycle C_m , $1 \leq i \leq m$, then the resultant graph is $(C_m \odot K_1)$. Then $(C_m \odot K_1) \cup P_n$ is a disconnected graph with $2m + n - 1$ edges and $2m + n$ vertices.

Case 1: n is odd

Define a bijection $\varphi: V \cup E \rightarrow \{1, 2, 3, \dots, 4m + 2n + 1\}$ such that

$$\varphi(u_i) = i, 1 \leq i \leq m-1$$

$$\varphi(v_i) = m + i, 1 \leq i \leq m-1$$

$$\varphi(u_n) = m; \varphi(v_n) = 2m$$

$$\varphi(w_i) = \begin{cases} 2m + \frac{i+1}{2}, & 1 \leq i \leq n, i \text{ is odd} \\ 2m + \frac{n+i+1}{2}, & 1 \leq i \leq n, i \text{ is even} \end{cases}$$

$$\varphi(u_iu_{i+1}) = 2m + 2n + 2i, 1 \leq i \leq m-1$$

$\varphi(u_iv_i) = 2m + 2n + 2i - 1, 1 \leq i \leq m$
 $\varphi(w_iw_{i+1}) = 2m + n + i, 1 \leq i \leq n - 1$ and $\varphi(u_nu_1) = 2m + 2n$.
Using this labeling, for each edge $uv \in E[(C_m \odot K_1) \cup P_n]$, $|\varphi(u) - \varphi(uv) + \varphi(v)|$ will form any one of the constants $c_1 = |1 - 2m - 2n|, c_2 = |1 - m - 2n|$ and $c_3 = \left\lfloor \frac{3-n+4m}{2} \right\rfloor$. Therefore, the disconnected graph $(C_m \odot K_1) \cup P_n$ admits an edge trimagic graceful labeling for n is odd.

Case 2: n is even

Define a bijection $\varphi: VUE \rightarrow \{1, 2, 3, ..., 4m + 2n + 1\}$ such that
 $\varphi(u_i) = i, 1 \leq i \leq m - 1$
 $\varphi(v_i) = m + i, 1 \leq i \leq m - 1$
 $\varphi(u_n) = m; \varphi(v_n) = 2m$

$$\varphi(w_i) = \begin{cases} 2m + \frac{i+1}{2}, 1 \leq i \leq n, i \text{ is odd} \\ 2m + \frac{n+i}{2}, 1 \leq i \leq n, i \text{ is even} \end{cases}$$
 $\varphi(u_iu_{i+1}) = 2m + 2n + 2i, 1 \leq i \leq m - 1$
 $\varphi(u_iv_i) = 2m + 2n + 2i - 1, 1 \leq i \leq m$
 $\varphi(w_iw_{i+1}) = 2m + n + i, 1 \leq i \leq n - 1$ and $\varphi(u_nu_1) = 2m + 2n$.
Using this labeling, for each edge $uv \in E[(C_m \odot K_1) \cup P_n]$, $|\varphi(u) - \varphi(uv) + \varphi(v)|$ will form any one of the constants $c_1 = |1 - 2m - 2n|, c_2 = |1 - m - 2n|$ and $c_3 = \left\lfloor \frac{2-n+4m}{2} \right\rfloor$. Therefore, the disconnected graph $(C_m \odot K_1) \cup P_n$ admits an edge trimagic graceful labeling for n is even.

We proved that the disconnected graph $(C_m \odot K_1) \cup P_n$ admits an edge trimagic graceful labeling. Since $(C_m \odot K_1) \cup P_n$ has $4m + 2n + 1$ vertices and these $4m + 2n + 1$ vertices have labels $1, 2, ..., 4m + 2n + 1$ for all values of n and m , the graph $(C_m \odot K_1) \cup P_n$ is a super edge trimagic graceful.

Example 2.2: An edge trimagic graceful labeling of the graph $(C_5 \odot K_1) \cup P_6$ and the graph $(C_6 \odot K_1) \cup P_7$ are given in figure 1 and figure 2 respectively.

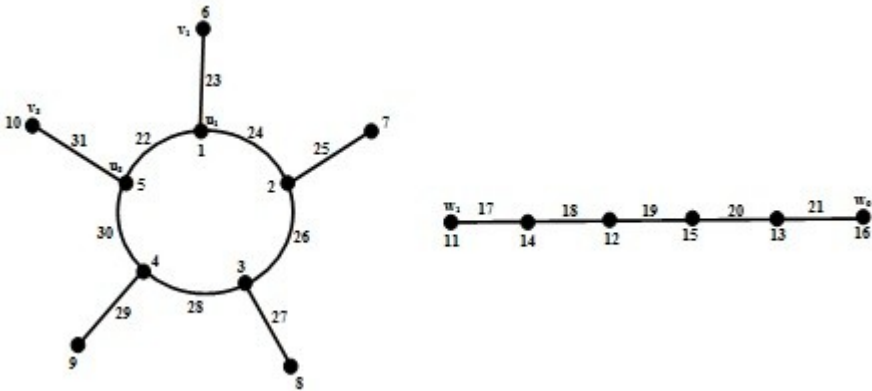


Figure 1: $(C_5 \odot K_1) \cup P_6$ with edge trimagic graceful constants $c_1 = 21, c_2 = 16$ and $c_3 = 8$

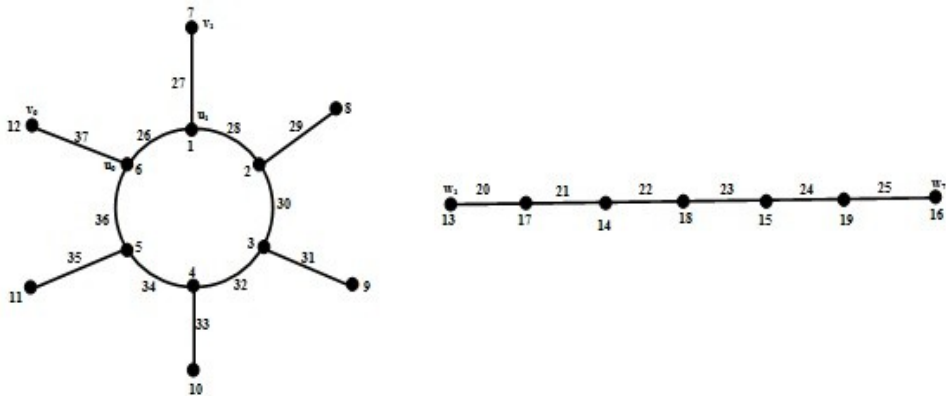


Figure 2: $(C_6 \odot K_1) \cup P_7$ with edge trimagic graceful constants $c_1 = 25, c_2 = 19$ and $c_3 = 10$

Theorem 2.3: The graph nP_3 admits an edge trimagic graceful labeling for all n .

Proof: Let $V(nP_3) = \{u_i, v_i, w_i / 1 \leq i \leq n\}$ be the vertex set, and $E(nP_3) = \{u_i v_i, v_i w_i / 1 \leq i \leq n\}$ be the edge set of the disconnected graph nP_3 . Then the disconnected graph nP_3 has $3n$ vertices and $2n$ edges. Define a bijection $\varphi: V \rightarrow \{1, 2, 3, \dots, 5n\}$ such that

$$\varphi(u_1) = 1; \varphi(u_{i+1}) = i + 1, 1 \leq i \leq n - 1$$

$$\varphi(v_1) = n + 1; \varphi(v_{i+1}) = n + i + 1, 1 \leq i \leq n - 1$$

$$\varphi(w_i) = 2n + i, 1 \leq i \leq n; \varphi(u_1 v_1) = 5n; \varphi(v_1 w_1) = 5n - 1$$

$$\varphi(u_{i+1} v_{i+1}) = 3n + 2i - 1, 1 \leq i \leq n - 1 \text{ and } \varphi(v_{i+1} w_i) = 3n + 2i, 1 \leq i \leq n - 1$$

Using this labeling, for each edge $uv \in E(nP_3)$, $|\varphi(u) - \varphi(uv) + \varphi(v)|$ will form any one of the constants $c_1 = |2 - 4n|$, $c_2 = |3 - 2n|$ and $c_3 = |2|$. Therefore, the disconnected graph nP_3 admits an edge trimagic graceful labeling for all n .

Since the disconnected graph nP_3 has $3n$ vertices and these $3n$ vertices have labels $1, 2, \dots, 3n$ for all n , the disconnected graph nP_3 is a super edge trimagic graceful graph.

Example 2.4: A super edge trimagic graceful labeling of $5P_3$ is given in figure 3.

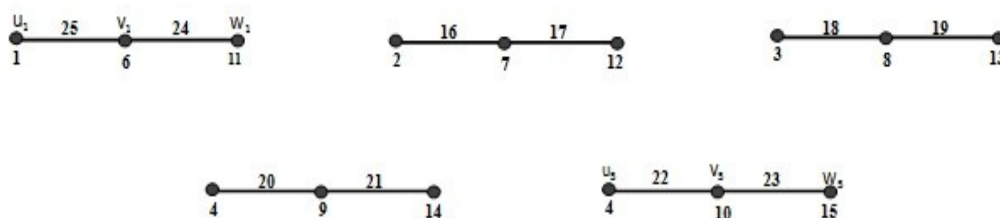


Figure 3: $5P_3$ with edge trimagic graceful constants $c_1 = 18$, $c_2 = 7$ and $c_3 = 2$.

Theorem 2.5: The graph $K_{1,l} \cup K_{1,m} \cup K_{1,n}$ admits an edge trimagic graceful labeling for all values of l, m and n .

Proof: Let $V(K_{1,l} \cup K_{1,m} \cup K_{1,n}) = \{u_i, v_j, w_k / 1 \leq i \leq l+1; 1 \leq i \leq m+1, 1 \leq i \leq n+1\}$ be the vertex set, and $E(K_{1,l} \cup K_{1,m} \cup K_{1,n}) = \{u_1 u_i, v_1 v_j, w_1 w_k / 2 \leq i \leq l+1; 2 \leq i \leq m+1, 2 \leq i \leq n+1\}$ be the edge set of the disconnected graph $K_{1,l} \cup K_{1,m} \cup K_{1,n}$. Then the disconnected graph $K_{1,l} \cup K_{1,m} \cup K_{1,n}$ has $l + m + n + 3$ vertices and $l + m + n$ edges.

Case 1: l, n are odd and m is even.

Define a bijection $\varphi: V \rightarrow \{1, 2, 3, \dots, 2l + 2m + 2n + 3\}$ such that

$$\varphi(u) = 1, \varphi(v) = 2, \varphi(w) = 3$$

$$\varphi(u_i) = 3 + i, 1 \leq i \leq l; \varphi(uu_i) = l + m + n + i + 3, 1 \leq i \leq l$$

$$\varphi(v_j) = m + j + 2, 1 \leq j \leq m; \varphi(w_k) = 2n + k, 1 \leq k \leq n$$

$$\varphi(vv_j) = l + m + 2n + j + 1, 1 \leq j \leq m$$

$$\text{and } \varphi(ww_k) = 2l + m + 2n + k + 2, 1 \leq k \leq n.$$

Using this labeling, for each edge $uv \in E(K_{1,l} \cup K_{1,m} \cup K_{1,n})$, $|\varphi(u) - \varphi(uv) + \varphi(v)|$ will form any one of the constants $c_1 = |1 - (l + m + n)|$, $c_2 = |3 - (l + 2n)|$ and $c_3 = |1 - (2l + m)|$. Therefore, the disconnected graph $K_{1,l} \cup K_{1,m} \cup K_{1,n}$ admits an edge trimagic graceful labeling for l, n are odd and m is even.

Case 2: l, m and n are odd.

Define a bijection $\varphi: V \rightarrow \{1, 2, 3, \dots, 2l + 2m + 2n + 3\}$ such that

$$\varphi(u) = 1, \varphi(v) = 2, \varphi(w) = 3$$

$$\varphi(u_i) = 3 + i, 1 \leq i \leq l; \varphi(v_j) = m + j + 1, 1 \leq j \leq m$$

$$\varphi(w_k) = m + n + k - 1, 1 \leq k \leq n; \varphi(uu_i) = l + m + n + i + 3, 1 \leq i \leq l.$$

$$\varphi(vv_j) = l + 2m + n + j + 1, 1 \leq j \leq m \text{ and } \varphi(ww_k) = l + 2m + 2n + k - 1, 1 \leq k \leq n.$$

Using this labeling, for each edge $uv \in E(K_{1,l} \cup K_{1,m} \cup K_{1,n})$, $|\varphi(u) - \varphi(uv) + \varphi(v)|$ will form any one of the constants $c_1 = |1 - (l + m + n)|$, $c_2 = |2 - (l + m + n)|$ and $c_3 = |3 - (l + m + n)|$. Therefore, the disconnected graph $K_{1,l} \cup K_{1,m} \cup K_{1,n}$ admits an edge trimagic graceful labeling for l, m and n are odd.

Case 3: l, m and n are even.

Define a bijection $\varphi: V \rightarrow \{1, 2, 3, \dots, 2l + 2m + 2n + 3\}$ such that

$$\varphi(u) = 1, \varphi(v) = 2, \varphi(w) = 3$$

$$\varphi(u_i) = 3 + i, 1 \leq i \leq l; \varphi(v_j) = m + j, 1 \leq j \leq m$$

$$\varphi(w_k) = l + n + k, 1 \leq k \leq n; \varphi(uu_i) = l + m + n + i + 3, 1 \leq i \leq l.$$

$$\varphi(vv_j) = 2l + m + n + j + 3, 1 \leq j \leq m \text{ and } \varphi(ww_k) = l + 2m + 2n + k - 1, 1 \leq k \leq n.$$

Using this labeling, for each edge $uv \in E(K_{1,l} \cup K_{1,m} \cup K_{1,n})$, $|\varphi(u) - \varphi(uv) + \varphi(v)|$ will form any one of the constants $c_1 = |1 - (l + m + n)|$, $c_2 = |-2l - n|$ and $c_3 = |5 - 2m - n|$. Therefore, the disconnected graph $K_{1,l} \cup K_{1,m} \cup K_{1,n}$ admits an edge trimagic graceful labeling for l, m and n are even.

Case 4: For all other cases (l, m are even and n is odd (or) m, n are odd and l is even (or) l, n are even and m is odd (or) l, m are odd and n is even (or) l is odd and m, n are even)

Define a bijection $\varphi: V \cup E \rightarrow \{1, 2, 3, \dots, 2l + 2m + 2n + 3\}$ such that

$$\begin{aligned} \varphi(u) &= 1, \varphi(v) = 2, \varphi(w) = 3 \\ \varphi(u_i) &= 3 + i, 1 \leq i \leq l; \varphi(v_j) = l + j + 3, 1 \leq j \leq m \\ \varphi(w_k) &= l + m + k + 3, 1 \leq k \leq n; \varphi(uu_i) = l + m + n + i + 3, 1 \leq i \leq l. \\ \varphi(vv_j) &= 2l + m + n + j + 3, 1 \leq j \leq m \\ \text{and } \varphi(ww_k) &= 2l + 2m + n + k + 3, 1 \leq k \leq n \end{aligned}$$

Using this labeling, for each edge $uv \in E(K_{1,l} \cup K_{1,m} \cup K_{1,n})$, $|\varphi(u) - \varphi(uv) + \varphi(v)|$ will form any one of the constants $c_1 = |1 - (l + m + n)|$, $c_2 = |2 - (l + m + n)|$ and $c_3 = |3 - (l + m + n)|$. Therefore, the disconnected graph $K_{1,l} \cup K_{1,m} \cup K_{1,n}$ admits an edge trimagic graceful labeling for all other cases such as l, m are even and n is odd, m, n are odd and l is even, l, n are even and m is odd, l, m are odd and n is even and l is odd and m, n are even. Hence the theorem follows from the above all four cases.

We proved the disconnected graph $K_{1,l} \cup K_{1,m} \cup K_{1,n}$ admits an edge trimagic graceful labeling. Since the disconnected graph $K_{1,l} \cup K_{1,m} \cup K_{1,n}$ has $l + m + n + 3$ vertices and these vertices have labels $1, 2, \dots, l + m + n + 3$, the disconnected graph $K_{1,l} \cup K_{1,m} \cup K_{1,n}$ is a super edge trimagic graceful graph.

Example 2.6: A super edge trimagic graceful labeling of $K_{1,7} \cup K_{1,9} \cup K_{1,11}$ is given in figure 4.

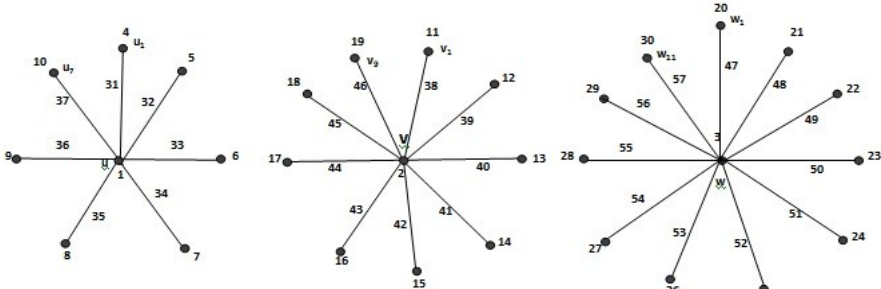


Figure 4: $K_{1,7} \cup K_{1,9} \cup K_{1,11}$ with edge trimagic graceful constants $c_1=26, c_2=25$ and $c_3=24$.

Theorem 2.7: The graph nC_4 admits an edge trimagic graceful labeling for all n .

Proof: Let $V(nC_4) = \{w_i, x_i, y_i, z_i / 1 \leq i \leq n\}$ be the vertex set and $E(nC_4) = \{w_i x_i, x_i z_i, y_i z_i, y_i w_i / 1 \leq i \leq n\}$ be the edge set of the disconnected graph nC_4 . Then the graph nC_4 has $4n$ vertices and $4n$ edges.

Define a bijection $\varphi: V \cup E \rightarrow \{1, 2, \dots, 8n\}$ such that

$$\begin{aligned} \varphi(w_i) &= i, 1 \leq i \leq n; \varphi(x_i) = n + i, 1 \leq i \leq n \\ \varphi(y_i) &= 2n + i, 1 \leq i \leq n; \varphi(z_i) = 3n + i, 1 \leq i \leq n. \\ \varphi(w_i x_i) &= 4n + i - 1, 1 \leq i \leq n; \varphi(x_i y_i) = 6n + 2i, 1 \leq i \leq n \\ \varphi(y_i z_i) &= 6n + 2i - 1, 1 \leq i \leq n \text{ and } \varphi(z_i w_i) = 4n + 2i, 1 \leq i \leq n. \end{aligned}$$

Using this labeling we get, for each edge $uv \in E(nC_4)$, $|\varphi(u) - \varphi(uv) + \varphi(v)|$ will form any one of the constants $c_1 = |1 - 3n|$, $c_2 = |-2n|$ and $c_3 = |1 - n|$. Hence the graph nC_4 is an edge trimagic graceful for all values of n .

We proved the graph nC_4 admits an edge trimagic graceful labeling. The labeling given in the proof of theorem 2.7, the vertices get labels $\varphi(w_i) = i, \varphi(x_i) = n + i, \varphi(y_i) = 2n + i, \varphi(z_i) = 3n + i$, for all $1 \leq i \leq n$. Since the graph nC_4 has $4n$ vertices and these $4n$ vertices have labels $1, 2, \dots, 4n$. Hence, the graph nC_4 is a super edge trimagic graceful graph.

Example 2.8: An edge trimagic graceful labeling of $5C_4$ is given in figure 5.

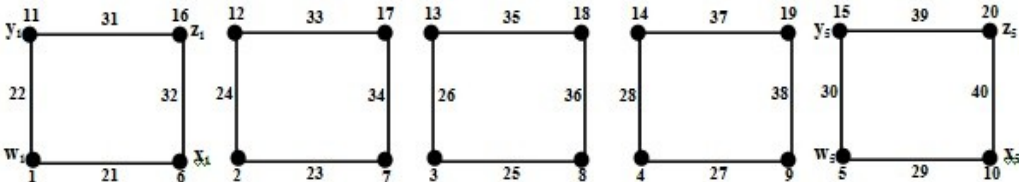


Figure 5: $5C_4$ with the edge trimagic graceful constants $c_1=14, c_2=10$ and $c_3=4$

Theorem 2.9: The disconnected graph $(C_m \odot K_1) \cup C_n$ admits an edge trimagic graceful labeling for all values of m and for odd n .

Proof: Let $V[(C_m \odot K_1) \cup C_n] = \{u_i, v_i, w_j, 1 \leq i \leq m, 1 \leq j \leq n\}$ be the vertex set, and $E[(C_m \odot K_1) \cup C_n] = \{u_i v_i, u_i u_{i+1}, w_j w_{j+1} / 1 \leq i \leq m, 1 \leq j \leq n\}$ be the edge set of the graph $(C_m \odot K_1) \cup C_n$. Then the graph $(C_m \odot K_1) \cup C_n$ has $2m + n$ vertices and $2m + n$ edges.

Case 1: m is odd

Define a bijection $\varphi: V \cup E \rightarrow \{1, 2, 3, \dots, 4m + 2n\}$ such that

$$\varphi(u_i) = i, 1 \leq i \leq m; \varphi(v_i) = m + i, 1 \leq i \leq m$$

$$\varphi(w_j) = \begin{cases} \frac{2m+2n+j-3}{2}, & 1 \leq i \leq n, j \text{ is odd} \\ \frac{2m+3n+j-3}{2}, & 1 \leq i \leq n, j \text{ is even} \end{cases}$$

$$\varphi(u_i u_{i+1}) = 2m + 2n + 2i + 1, 1 \leq i \leq m - 1; \varphi(u_1 u_m) = 2m + 2n + 1$$

$$\varphi(u_i v_i) = 2m + 2n + 2i, 1 \leq i \leq m; \varphi(w_1 w_n) = 2m + 2n + 2i - 1$$

$$\text{and } \varphi(w_i w_{i+1}) = 2m + n + i + 1, 1 \leq j \leq n - 1$$

Using this labeling, for each edge $uv \in E[(C_m \odot K_1) \cup C_n]$, $|\varphi(u) - \varphi(uv) + \varphi(v)|$ will form any one of the constants $c_1 = |-2m - 2n|$, $c_2 = |-2n - m|$ and $c_3 = \left| \frac{3n-7}{2} \right|$. Therefore, the graph $(C_m \odot K_1) \cup C_n$ admits an edge trimagic graceful labeling for odd m .

Case 2: m is even

Define a bijection $\varphi: V \cup E \rightarrow \{1, 2, 3, \dots, 4m + 2n\}$ such that

$$\varphi(u_i) = i, 1 \leq i \leq m; \varphi(v_i) = m + i, 1 \leq i \leq m$$

$$\varphi(w_j) = \begin{cases} 2m + \frac{i+1}{2}, & 1 \leq i \leq n, i \text{ is odd} \\ 2m + \frac{n+i+1}{2}, & 1 \leq i \leq n, i \text{ is even} \end{cases}$$

$$\varphi(u_i u_{i+1}) = 2m + 2n + 2i + 1, 1 \leq i \leq m - 1; \varphi(u_1 u_m) = 2m + 2n + 1$$

$$\varphi(u_i v_i) = 2m + 2n + 2i, 1 \leq i \leq m; \varphi(w_1 w_n) = 2m + n + 1$$

$$\text{and } \varphi(w_i w_{i+1}) = 2m + n + j + 1, 1 \leq j \leq n - 1$$

Using this labeling, for each edge $uv \in E[(C_m \odot K_1) \cup C_n]$, $|\varphi(u) - \varphi(uv) + \varphi(v)|$ will form any one of the constants $c_1 = |-2m - 2n|$, $c_2 = |-m - 2n|$ and $c_3 = \left| \frac{4m+1-n}{2} \right|$. Therefore, the graph $(C_m \odot K_1) \cup C_n$ admits an edge trimagic graceful labeling for even m .

Hence, the theorem follows from case 1 and case 2.

Since the graph $(C_m \odot K_1) \cup C_n$ has $2m + n$ vertices and these $2m + n$ vertices have labels $1, 2, \dots, 2m + n$. Hence, the graph $(C_m \odot K_1) \cup C_n$ is a super edge trimagic graceful graph.

Example 2.10: An edge trimagic graceful labeling of $(C_5 \odot K_1) \cup C_7$ and $(C_6 \odot K_1) \cup C_9$ are given in figure 6 and figure 7.

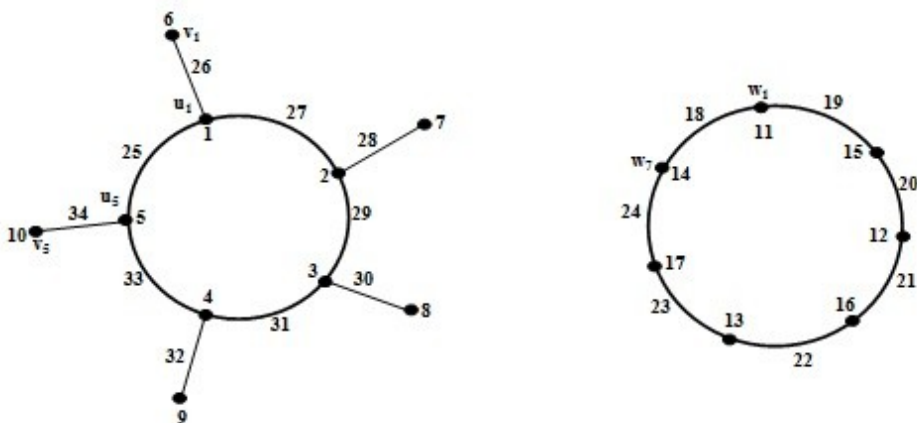


Figure 6: $(C_5 \odot K_1) \cup C_7$ with edge trimagic graceful constants $c_1 = 24$, $c_2 = 19$ and $c_3 = 7$.

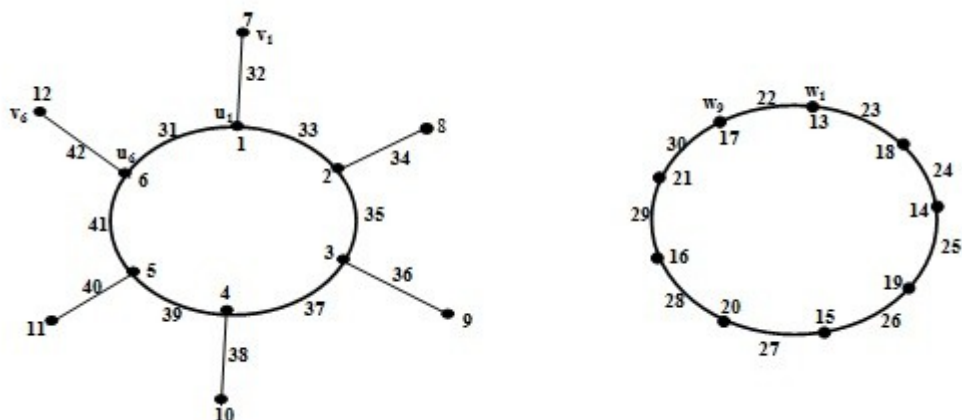


Figure 7: $(C_6 \odot K_1) \cup C_9$ with edge trimagic graceful constants $sc_1 = 30$, $c_2 = 24$ and $c_3 = 8$.

CONCLUSION

In this paper, we proved that the disconnected graphs $(C_m \odot K_1) \cup P_n$, nP_3 , $K_{1,l} \cup K_{1,m} \cup K_{1,n}nC_4$ and $(C_m \odot K_1) \cup C_n$ are edge trimagic graceful graphs, using these ideas we can find more Edge Trimagic Graceful labeling of some more disconnected graphs.

ACKNOWLEDGEMENT

I would like to express my special thanks of gratitude to my Guide Dr. M. Regees and my Co – Guide Dr. T. Shyla Isac Mary for their able guidance and support for my Research. And I like to thank the editor and the referees for their exceptionally helpful comments during the review process.

REFERENCES

1. J. Baskar Babujee, "On Edge Bimagic labeling", Journal of Combinatorics Information & System Sciences, Vol. 28, No. (1– 4) (2004) 239 – 244.
2. J. Baskar Babujee, R. Jagadesh, "Super Edge Bimagic Labeling for Disconnected Graphs", International Journal of Mathematics Engineering Sciences, Vol. 2 (2008), No. 2, pp. 171 – 175.
3. S. Babitha, A. Amarajothi, J. Baskar Babujee, "Magic and Bimagic Labeling for Disconnected Graphs", International Journal of Mathematics Trends and Technology, Vol. 3 (2012), No. 2, pp. 86 – 90.
4. Golomb, S. W., How to number a graph, Graph Theory and Computing (ed. R.C. Read), Academic Press, New York, 1977, pp. 23–37.
5. Joseph A. Gallian, "A Dynamic Survey of graph Labeling of Some Graphs", The Electronic Journal of Combinatorics (2017), # DS6.
6. A. Kotzig and A. Rosa, "Magic Valuations of finite graphs", Canad. Math. Bull., vol.13 (1970) 415 – 416.
7. G. Marimuthu and M. Balakrishnan, "Super Edge Magic graceful Graphs", Information sciences 287(2014), 140 – 151.
8. C. Jayasekaran, M. Regees and, C Davidraj "Edge Trimagic labeling of some Graphs" International Journal for Combinatorial Graph theory and applications – 6 (2), (2013), pp. 175 – 186.
9. M. Regees and C. Jayasekaran, "Edge Trimagic Total Labeling for Disconnected Graphs", International Journal of Mathematics Trends and Technology, Vol. 6 (2014), pp. 44 – 53.